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Balanced equi-n-squares

We define a d-balanced equi-n-square $L = (l_{ij})$, for some divisor d of n, as an $n \times n$ matrix containing symbols from $\mathbb{Z}_n$ in which any symbol that occurs in a row or column, occurs exactly d times in that row or column. We show how to construct a d-balanced equi-n-square from a partition of a Latin square of order n into $d \times (n/d)$ subrectangles. In design theory, L is equivalent to a decomposition of $K_{n,n}$ into d-regular spanning subgraphs of $K_{n/d,n/d}$. We also study when L is diagonally cyclic, defined as when $l_{(i+1)(j+1)} = l_{ij} + 1$ for all $i, j \in \mathbb{Z}_n$, which corresponds to cyclic such decompositions of $K_{n,n}$ (and thus α-labellings).

We identify necessary conditions for the existence of (a) d-balanced equi-n-squares, (b) diagonally cyclic d-balanced equi-n-squares, and (c) Latin squares of order n which partition into $d \times (n/d)$ subrectangles. We prove the necessary conditions are sufficient for arbitrary fixed $d \geq 1$ when n is sufficiently large, and we resolve the existence problem completely when $d \in \{1, 2, 3\}$.

Along the way, we identify a bijection between α-labellings of d-regular bipartite graphs and, what we call, d-starters: matrices with exactly one filled cell in each top-left-to-bottom-right unbroken diagonal, and either d or 0 filled cells in each row and column. We use d-starters to construct diagonally cyclic d-balanced equi-n-squares, but this also gives new constructions of α-labellings.