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Singularity Formation in the Deterministic and Stochastic Fractional Burgers Equation.

Motivated by the results concerning the regularity of solutions to the fractional Navier-Stokes system and questions about the influence of noise on the formation of singularities in hydrodynamic models, we have explored these two problems in the context of the fractional 1D Burgers equation. First, we performed highly accurate numerical computations to characterize the dependence of the blow-up time on the fractional dissipation exponent in the supercritical regime. The problem was solved numerically using a pseudospectral method where integration in time was performed using a hybrid method combining the Crank-Nicolson and a three-step Runge-Kutta technique. A highlight of this approach is automated resolution refinement. The blow-up time was estimated based on the time evolution of the enstrophy (H^1 seminorm) and the width of the analyticity strip. The consistency of the obtained blow-up times was verified in the limiting cases. In the second part of the thesis we considered the fractional Burgers equation in the presence of suitably colored additive noise. This problem was solved using a stochastic Runge-Kutta method where the stochastic effects were approximated using a Monte-Carlo method. Statistic analysis of ensembles of stochastic solutions obtained for different noise magnitudes indicates that as the noise amplitude increases the distribution of blow-up times becomes non-Gaussian. In particular, while for increasing noise levels the mean blow-up time is reduced as compared to the deterministic case, solutions with increased existence time also become more likely.