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The vanishing of L-series and the Okada space

If f is a complex-valued arithmetical function with period N, we associate the L-series

$$L(s,f) := \sum_{n=1}^{\infty} \frac{f(n)}{n^s}.$$

It is easy to see that this series converges for $\Re(s) > 1$ and admits an analytic continuation to the entire complex plane except at $s = 1$ where it has a simple pole with residue

$$\frac{1}{N}\sum_{a=1}^{N} f(a).$$

Thus, $L(1,f)$ is finite if and only if the residue is zero, which we shall assume. The Okada space consists of all such functions f for which $L(1,f) = 0$. We construct an explicit basis for this vector space. As a consequence, we are able to derive results about $\mathbb{Q}$-linear relations among special values of the digamma function at rational arguments. This is joint work with Siddhi Pathak.