
PHILIP ERNST, Rice Statistics

Quickest real-time detection of a Brownian coordinate drift

Consider the motion of a Brownian particle in two or more dimensions, whose coordinate processes are standard Brownian motions with zero drift initially, and then at some random/unobservable time, one of the coordinate processes gets a (known) non-zero drift permanently. Given that the position of the Brownian particle is being observed in real time, the problem is to detect the time at which a coordinate process gets the drift as accurately as possible. We solve this problem in the most uncertain scenario when the random/unobservable time is (i) exponentially distributed and (ii) independent from the initial motion without drift. The solution is expressed in terms of a stopping time that minimizes the probability of a false early detection and the expected delay of a missed late detection. To our knowledge this is the first time that such a problem has been solved exactly in the literature. This is joint work with Goran Peskir (University of Manchester).