The Mathematics behind Quantum Information Science
Les mathématiques derrière la science de l'information quantique
(Org: **Nathaniel Johnston** (Mount Allison) and/et **Sarah Plosker** (Brandon))

SABINE BURGDORF, University of Konstanz
Quantum correlations and optimization

Entanglement is one of the key features of quantum mechanics, e.g., creating (bipartite) correlations which cannot be obtained classically. There are basically two mathematical models to describe bipartite quantum correlations: via commuting operators and via tensor products of operators. We will present conic descriptions of these sets, which allows a broader use of (real) algebraic methods for a better understanding of quantum correlations. We will discuss some of these aproaches mostly related to approximation methods from conic or polynomial optimization.

ERIC CHITAMBAR, University of Illinois at Urbana-Champaign
Playing Mermin's Game with Nonlocal Resources

In a popular science article [Am. J. of Phys. 49, 940 (1981)], N.D. Mermin presented a conceptually simple demonstration of quantum nonlocality. The phenomenon is described using a pair of three-input/two-output boxes that are constrained to have identical outputs whenever the same inputs are chosen. The game then consists in trying to maximize the probability of differing outputs whenever different inputs are chosen. In this talk I will describe a variant of Mermin's game in which the constraint of identical outputs is relaxed. When the constraint is completely removed the CHSH is recovered, while in general the largest quantum advantage is shown to scale linearly in the relaxation parameter ϵ. We then consider playing Mermin's game with nonlocal boxes, or PR-boxes. We show that the optimal score in Mermin's game with one PR-box is $1/6$ while with two PR-boxes the game can be won perfectly.

JASON CRANN, Carleton University
State convertibility in the von Neumann algebra framework.

Nielsen characterized the convertibility of two finite-dimensional bipartite pure states via local operations and classical communication (LOCC) using majorization. This important result, which has seen many applications in quantum information, describes the LOCC-transfer of entanglement between bipartite pure states. In this talk, we present a version of Nielsen's theorem in the commuting operator framework using a generalized class of LOCC operations and the theory of majorization in von Neumann algebras. As a corollary, we obtain an operational interpretation of maximal entanglement relative to von Neumann factors of type II_1. This is joint work with David Kribs, Rupert Levene and Ivan Todorov.

GILAD GOUR, University of Calgary
Theories of Dynamical Quantum Resources

A common theme in Chemistry, Thermodynamics, and Information Theory is how one type of resource – be it chemicals, heat baths, or communication channels – can be used to produce another. These processes of conversion and their applications are studied under the general heading of "resource theories". While resource theories use a wide range of sophisticated and apparently unrelated mathematical techniques, there is also an emerging general mathematical framework which seems to underpin all of them. In this talk, I will introduce some of these common mathematical structures that appear in resource theories, particularly those appearing in resource theories of quantum processes.

SAM HARRIS, University of Waterloo
Separating the matrix-valued bipartite correlation sets

In recent years, there has been much study devoted to various sets of quantum bipartite correlations in a finite-input, finite-output system. These sets are often denoted by $C_t(m,k)$, where m is the number of inputs, k is the number of outputs, and t represents the model that is being used. Some of the most notable models are the finite-dimensional (tensor product) model ($t=q$) and the tensor product model ($t=qs$). Thanks to recent work of W. Slofstra, it is known that $C_{qs}(m,k)$ is not a closed set if m and k are large enough. Recent work of A. Coladangelo and J. Stark shows that $C_q(5,3) \neq C_{qs}(5,3)$. In this talk, we consider a matrix-valued generalization of these sets, denoted $C_t^{(n)}(m,k)$, where Alice and Bob have access to n (orthonormal) states instead of just 1. We show that there is some $n \leq 13$ such that, whenever $m,k \geq 2$ and $(m,k) \neq (2,2)$, we have that $C_q^{(n)}(m,k) \neq C_{qs}^{(n)}(m,k)$ and that $C_{qs}^{(n)}(m,k)$ is not a closed set. This is based on joint work with Li Gao and Marius Junge.

NATHANIEL JOHNSTON, Mount Allison University
Pairwise Completely Positive Matrices and Quantum Entanglement

We introduce a generalization of the set of completely positive matrices that we call "pairwise completely positive" (PCP) matrices. These are pairs of matrices that share a joint decomposition so that one of them is necessarily positive semidefinite while the other one is necessarily entrywise non-negative. We explore basic properties of these matrix pairs and develop several testable necessary and sufficient conditions that help determine whether or not a pair is PCP. We then establish a connection with quantum entanglement by showing that determining whether or not a pair of matrices is pairwise completely positive is equivalent to determining whether or not a certain type of quantum state, called a conjugate local diagonal unitary invariant state, is separable. Many of the most important quantum states in entanglement theory are of this type, including isotropic states, mixed Dicke states (up to partial transposition), maximally correlated states, as well as the central states of interest in the absolute separability problem.

RUPERT LEVENE, University College Dublin
Schur multipliers and mixed unitary maps

We consider the tensor product of the completely depolarising channel on $d \times d$ matrices with the map of Schur multiplication by a $k \times k$ correlation matrix and characterise, via matrix theory methods, when such a map is a mixed (random) unitary channel. When $d=1$, this recovers a result of O'Meara and Pereira, and for larger d is equivalent to a result of Haagerup and Musat that was originally obtained via the theory of factorisation through von Neumann algebras. We obtain a bound on the distance between a given correlation matrix for which this tensor product is nearly mixed unitary and a correlation matrix for which such a map is exactly mixed unitary. This bound allows us to give an elementary proof of another result of Haagerup and Musat about the closure of such correlation matrices without appealing to the theory of von Neumann algebras.

This is joint work with Sam Harris, Vern Paulsen, Sarah Plosker and Mizanur Rahaman.

JEREMY LEVICK,
Factorizable Quantum Channels and Linear Matrix Inequalities

We find a connection between the existence of a factorization of a quantum channel through the algebra $M_N(\mathbb{C})$ and the existence of low-rank solutions to certain linear matrix inequalities. Using this, we show that if a quantum channel is factorized by a direct integral of factors, it must lie in the convex hull of quantum channels which are factorized respectively by the factors in the direct integral. We use this to characterize some non-trivial extreme points in the set of factorizable quantum channels and give a class of examples.

COMFORT MINTAH,
Operator algebras and quantum one-way LOCC state distinguishability

We study the physical description of Quantum Local Operations and Classical Communications (LOCC) and its schematics. We restrict ourselves to one-way LOCC (one of the schemes of LOCC) and discuss detailed analysis in quantum information of recently derived operator relations. We indicate how operator structures such as operator systems and operator algebras naturally arise from these settings and make use of these structures to derive new result and new derivations of some established results in one-way LOCC. We compare perfect distinguishability of one-way LOCC versus arbitrary quantum operations and see how for several families of operators that appear jointly in matrix and operator theory and quantum information theory, the relations are equivalent.

SATISH PANDEY, Technion

VERN PAULSEN, Univerity of Waterloo
Constant Gap for Self-embezzlement

W. van Dam and P. Hayden proved that in the standard tensor model for representing bipartite quantum systems, it is impossible to catalytically produce an entangled state. But that as one allowed the dimensions of the state spaces to increase one could carry out this process to arbitrary precision. They referred to this process as "embezzlement" since if one knew the accuracy to which a third party could make observations, then one could "appear" to carry out an impossible task. Later in joint work with Cleve and Liu, we proved that one could not carry out this catalytic production of entangled states in the tensor model even if one allowed infinite dimensional state spaces, but one could carry it out exactly in the commuting model for bipartite systems. In this work we consider the task of not just producing any entangled state but producing the entangled state that is itself the catalyst. We prove that in finite dimensions, there is a constant gap, independent of the dimension, on how "nearly" one can carry out this task. We then prove that this task can be carried out exactly in infinite dimensions in the commuting model.

In this way we obtain a "task" that can be done in infinite dimensions but can not be done approximately in finite dimensions. This talk is based on joint work with B. Collins, R. Cleve and L. Liu.

RAJESH PEREIRA, University of Guelph
Quasiorthogonal algebras

Two unital subalgebras of a matrix algebra are said to be quasiorthogonal if their trace zero-subspaces are orthogonal in the trace inner product norm. We will explore some existence results for quasiorthogonal algebras and some of their mathematical properties. We will also discuss the application of quasiorthogonality to topics in quantum information (such as quantum error correction, quantum privacy and entanglement).

SARAH PLOSKER, Brandon University
The robustness of k-coherence

The degree to which a quantum state is in superposition with respect to a given orthonormal basis is called the coherence of the state. Numerous measures of coherence have been identified and studied recently. Here, we are interested in two separate generalizations of the robustness of coherence. We show that the two measures agree with each other when restricted to pure-state inputs by deriving an explicit closed expression.

MIZANUR RAHAMAN, University of Waterloo
A new bound on quantum Wielandt inequality

The Wielandt number for a primitive matrix with non-negative entries, is the minimum number of self-compositions needed so that all its entries become non-zero. In 2010, this concept has been generalized for trace preserving and completely positive maps (quantum channels). In this talk, I will establish a bound on quantum Wielnadt inequality for positive maps as opposed to quantum channels. This bound depends only on the dimension of the system the map is acting on and not on the map itself.

The techniques used to get this new bound provides a way to obtain improved bounds for this inequality for some specific classes of quantum channels. The motivation of this work is to provide an answer to a question raised by Sanz-Garcia-Wolf and Cirac who introduced the Wielandt inequality for quantum channels.

NEIL J. ROSS, Dalhousie University
A Characterization of Integral, Real, and Gaussian Clifford+T Operators

In 2012, Giles and Selinger showed that Clifford+T operators correspond to matrices of the form $U = (1/\sqrt{2})^k M$ where k is a nonnegative integer and M is a matrix over the ring $\mathbb{Z}[\omega]$. Here, we consider the operators that arise when one restricts M to be a matrix over a subring of $\mathbb{Z}[\omega]$. We focus on the subrings $\mathbb{Z}$, $\mathbb{Z}[\sqrt{2}]$, and $\mathbb{Z}[i]$, which define the integral, real, and Gaussian Clifford+T operators, respectively. We prove that these restricted Clifford+T operators correspond to circuits over well-known universal sets of quantum gates. Explicitly, we show that the integral Clifford+T operators are generated by the gate set $\{X, CX, CCX, H\}$, from which the real and Gaussian operators are obtained by adding the CH and S gate, respectively.

CARLO MARIA SCANDOLO, University of Calgary
Necessary and Sufficient Conditions on Measurements of Quantum Channels

Quantum supermaps are a higher-order generalization of quantum maps, taking quantum maps to quantum maps. It is known that any completely positive, trace non-increasing (CPTNI) map can be performed as part of a quantum measurement. By providing an explicit counterexample we show that, instead, not every quantum supermap sending a quantum channel to a CPTNI map can be realized in a measurement on quantum channels. We find that the supermaps that can be implemented in this way are exactly those transforming quantum channels into CPTNI maps even when tensored with the identity supermap.

JAMIE SIKORA, Perimeter Institute
Shadow Probabilities

Semidefinite optimization has proven to be a powerful tool in the study of quantum theory, with a wide array of applications. In this talk, I'll present a concept known as shadow prices which are quantities studied in mathematical economics, usually presented in the context of linear optimization. I'll discuss shadow prices in the context of semidefinite optimization and demonstrate how they appear in different areas of quantum theory, giving rise to shadow probabilities.