Mathematical Techniques for Analysing Quantum Structures and Materials
Techniques mathématiques pour l'analyse de structures et de matériaux quantiques

(Org: **Steven Rayan** and/et **Artur Sowa** (Saskatchewan))

GILAD GOUR, University of Calgary

Mathematical structures and features of quantum resource theories

A common theme in Chemistry, Thermodynamics, and Information Theory is how one type of resource – be it chemicals, heat baths, or communication channels – can be used to produce another. These processes of conversion and their applications are studied under the general heading of "resource theories". While resource theories use a wide range of sophisticated and apparently unrelated mathematical techniques, there is also an emerging general mathematical framework which seems to underpin all of them. In this talk, I will give an overview on the mathematical techniques and structure of quantum resource theories, with examples from resource theories of entanglement, asymmetry, quantum coherence, and quantum thermodynamics. I will end with several open problems.

ROBERT GREEN, University of Saskatchewan

Numerical Many Body Models for Synchrotron Spectroscopy of Quantum Materials

Significant research efforts are currently directed at the field of quantum materials, as many of these materials exhibit remarkable properties which may be suitable for next-generation device technologies. Synchrotron facilities, like the Canadian Light Source, are often utilized to study the properties of quantum materials via various forms of x-ray spectroscopy. However, interpreting the obtained spectroscopy data is often a significant challenge, as the connection from the fundamental quantum properties to the emergent spectral functions can be highly nontrivial. To this end, we design many-body quantum models which aim to capture the key properties of the materials while also having computable spectral functions which can be compared to experiment. In this talk, I'll introduce the models used and discuss strategic basis transformations and the numerical methods we employ. I will include recent results from studies of several highly interesting materials, including those exhibiting two-dimensional electron liquids and metal-insulator transitions. The concerted approach of synchrotron experiments and quantum many body models promises to be a key component of future work toward widespread quantum devices and other technologies utilizing quantum materials.

JOSEPH MACIEJKO, University of Alberta

Strongly interacting topological phases of matter

The discovery of topological band insulators in the mid-2000s, recognized in part by the 2016 Nobel Prize in Physics, has revolutionized condensed matter physics. In these materials, global properties of the quantum wavefunction are characterized by nontrivial topological invariants which distinguish homotopy classes of maps from momentum space to spaces of single-particle quantum Hamiltonians. This description however ignores interparticle interactions such as the electrostatic repulsion between electrons, which is nonetheless present in real materials. While weak interactions are not expected to significantly affect the topological classification of quantum materials, strong interactions have the potential to lead to novel topological phases beyond topological band insulators. In this talk I will discuss two examples of strongly interacting topological phases in 2+1 dimensions: a topologically nontrivial antiferromagnetic phase, and a symmetry-protected topological phase of fermions with no free-fermion counterpart.

CIHAN OKAY, University of British Columbia

Topology of quantum contextuality

Contextuality is a special feature of quantum systems. Originally it is expressed in the form of no-go theorems of Kochen-Specker, and violation of Bell inequalities. This fundamental property of quantum systems, which turns out to be responsible for speed-up in quantum computers, has been under intense investigation by the quantum computing community. In this work, joint with Daniel Sheinbaum, I will describe a topological approach to contextuality that uses classifying spaces, fundamental objects in algebraic topology. Physically relevant quantities are interpreted as classes in the cohomology and the twisted K-theory of the space.

NEIL J. ROSS, Dalhousie University
Number-Theoretic Methods in Quantum Compiling

Quantum compiling is concerned with the representation of general unitary operations by circuits built from some chosen set of quantum gates. The circuit representation of a unitary U is exact if the product of the gates composing the circuit is equal to U. The representation is approximate up to $\epsilon > 0$ if this product is at distance ϵ of U in the operator norm. In the last few years, the field of quantum compiling was rejuvenated by the introduction of methods from algebraic number theory. In particular, such number-theoretic methods were used to provide an optimal solution to the problem of approximating single-qubit unitaries using Clifford+T circuits. In this talk, I will present an efficient algorithm for the optimal approximation of single-qubit unitaries using Clifford+T circuits and discuss open problems in the field of quantum compiling.

ARTUR SOWA, University of Saskatchewan
Qubits, wavelets, fractals, bands

The traditional focus of Quantum Information Theory is structures comprising a finite number of qubits. However, it is also rewarding to study transfinite objects, such as infinite arrays of qubits. Most research on the physics of such structures relies upon one approximate technique or another. At the same time, it is desirable to collect examples of exactly solvable models, which rigorously capture how the functional properties of arrays (e.g. how they interact with modes of light) depend on their quantum state.

In the first part of my talk, I will discuss such a model (joint work with A. Zagoskin, ref.1). Our analysis directly involves multiresolution analysis, specifically the Haar basis. While applications of the Fourier transform in studies of spin systems are already classical, a quantum application of the Haar transform is, to our best knowledge, unprecedented. Also intriguing is an unexpected emergence of fractals in this very context. In the time remaining I will discuss some other connections between classical harmonic analysis and modern quantum theory, ref.2, and implications. In particular, I will demonstrate that band gaps can arise in a qubit-array Hamiltonian via a mechanism that does not involve a periodic potential.

References:

1. A. Sowa and A. Zagoskin, An exactly solvable quantum-metamaterial type model, Preprint, available via https://arxiv.org/abs/1902.05324
2. A. Sowa, Encoding spatial data into quantum observables, Preprint, available via http://arxiv.org/abs/1609.01712

RAY SPITERI, University of Saskatchewan
Quantum control for high-fidelity multi-qubit gates

Quantum control for error correction is critical for the practical use of quantum computers. We address quantum optimal control for single-shot multi-qubit gates by framing it as a feasibility problem for the Hamiltonian model that is then solved with standard global optimization software. Our approach yields faster high-fidelity (>99.99%) single-shot three-qubit-gate control than obtained previously, and it has also enabled us to solve the quantum-control problem for a fast high-fidelity four-qubit gate.

ALEX ZAGOSKIN, Loughborough University
Towards the qualitative theory of large quantum coherent structures

The impossibility of an efficient simulation of large enough quantum coherent systems by classical means, independently recognized by Feynman and Manin in early 1980s, launched the development of quantum computing and, more generally, the Second Quantum Revolution. It also remains the major obstacle for the development of quantum technologies 2.0, since the multiqubit structures which are being designed and fabricated at the moment are already too big to allow an efficient classical simulation, characterization and optimization, but too small and imperfect to serve as quantum computers capable of performing such a task. The situation is somewhat reminiscent of the aircraft development in the 20th century prior to the creation of computers powerful enough to solve the equations of hydrodynamics in realistic cases. I will use the analogy to discuss a possible strategy for circumventing the roadblock, which is based on the conjecture that there exist qualitatively different regimes of operation of large quantum coherent structures governed by a set of universal dimensionless parameters.