
JS LEMAY, University of Oxford

The Poincaré Lemma for Codifferential Categories with Antiderivatives

The Poincaré Lemma, named after the french mathematician Henri Poincaré, states that for a contractible manifold: a closed differential form is exact. In particular this implies that the de Rham complex of \mathbb{R}^n is contractible (or equivalently split exact). Similarly, the algebraic version of the Poincaré Lemma states that the algebraic de Rham complex (the one built from Kähler differentials) of a polynomial ring $\mathbb{R}[x_1, \dots, x_n]$ is contractible. In this talk we provide a Poincaré Lemma for codifferential categories with antiderivatives by showing that the de Rham complex of a free S-algebra is contractible, where S is the monad of the codifferential category. Taking S to be the free symmetric algebra monad results in the algebraic Poincaré Lemma, while taking S to be the free \mathcal{C}^∞ -ring monad gives the classical Poincaré Lemma.