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Smallest singular value of inhomogeneous random square matrices via double counting and random rounding

We show that the small ball behavior of random square matrices is optimal under minimal assumptions. An important step in the proof is showing that the “random normal” — random vector orthogonal to a collection of $(n-1)$ random vectors — has very good behavior, and its projection onto another random vector is not concentrated on any short interval. Previously, such result was known only under additional i.i.d. assumption, and the key technique leading to it was developed by Rudelson and Vershynin. Their approach, however, does not work without the i.i.d. requirement.

In order to show that the random normal is “good” we prove that the set of “bad” vectors is small: we construct a net on it of small cardinality. This net is a subset of a net on the sphere with simple lattice structure, and its construction relies on the method of random rounding. To show that the cardinality of this subset is small, we show that most of the vectors on a lattice are “good”, and therefore cannot be close to “bad” vectors. This key step is done via harmonic-analytic techniques from discrepancy theory. This is a joint work with K. Tikhomirov and R. Vershynin.