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Volume exhausting, T-equivariant symplectic embeddings of toric manifolds into regular coadjoint orbits

Let K be a compact connected Lie group and let $\mathcal{O}_\lambda$ denote the coadjoint orbit of K parameterized by an element λ in the positive Weyl chamber. In several cases, it is known that there exist dense symplectic embeddings of symplectic toric manifolds into $\mathcal{O}_\lambda$. If $K = U(n)$, then, for all λ, one obtains such embeddings from action-angle coordinates for the Gelfand-Zeitlin integrable systems. For arbitrary K compact and connected, if λ is a scalar multiple of an integral weight, then such embeddings can be constructed by toric degeneration and gradient-Hamiltonian flow (cf. Harada and Kaveh). It remains to study the case where λ is not a scalar multiple of an integral weight.

In current work with Anton Alekseev, Benjamin Hoffman, and Yanpeng Li, we show that for K semisimple and any regular coadjoint orbit $\mathcal{O}_\lambda$, one can construct a family of volume exhausting symplectic embeddings of toric manifolds into $\mathcal{O}_\lambda$. Moreover, these embeddings are equivariant with respect to a Hamiltonian action of a maximal torus of K. Our construction combines elements of Poisson-Lie theory, cluster algebras, and a scaling limit of Poisson structures called "tropicalization." In this talk I will endeavour to explain these results as well as our hopes for future work.