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Functional distribution monads and τ-additive measures

The article [1] defines a categorical framework for algebraic dualization processes that give rise to various spaces of measures, distributions, filters, closed subsets, compacta, and so forth. In this framework, the latter are all captured as instances of a general construction that begins with a $\mathcal{V}$-enriched algebraic category $\mathcal{A}$, with a suitable object to play the role of 'dualizer', and produces an associated monad on $\mathcal{V}$ called the *functional distribution monad*.

In this talk, we will show that by taking as $\mathcal{V}$ the category of convergence spaces and as $\mathcal{A}$ the category of convex spaces internal to $\mathcal{V}$, with the unit interval as dualizer, the induced functional distribution monad gives rise to the notion of τ-*additive* (or τ-*smooth*) *probability measure* on Tikhonov spaces. Thus the resulting monad captures a wide class of settings in topological measure theory, including not only Radon probability measures on locally compact spaces but also Borel probability measures on Polish spaces. In proving our result, we establish a connection between τ-additive measures and *continuous convergence*, and we prove integral representation theorems for τ-additive measures that are formulated in terms of the cartesian closed structure of the category of convergence spaces.

[1] R. B. B. Lucyshyn-Wright, Functional distribution monads in functional-analytic contexts. *Advances in Mathematics* 322 (2017), 806–860.