Cohomology – a link between numbers and geometry
Cohomologie - un lien entre les nombres et la géométrie
(Org: **Stefan Gille** (University of Alberta), **Jan Minac** (Western University) and/et **Kirill Zainoulline** (Univeristy of Ottawa / Université d'Ottawa))

ALEJANDRO ADEM, University of British Columbia
Free Finite Group Actions on Rational Homology Spheres

We use methods from group cohomology to describe those finite groups that can act freely and homologically trivially on a rational homology 3-sphere.

SUNIL CHEBOLU, Illinois State University
Groups with periodic cohomology

I will present the role of groups with periodic cohomology when studying the following problems.

1. When is the Tate cohomology functor faithful on the thick subcategory generated by k, the trivial representation of G?

2. When is the Tate cohomology of a finitely generated kG module finitely generated over the Tate cohomology of G with coefficients in k?

3. Which groups occur as the group of units of a ring?

This is based on joint projects with Jon Carlson, Jan Minac and Keir Lockridge.

LUCILE DEVIN, University of Ottawa
Divisbvility by p of $N_X(p)$

For any scheme X over $\mathbf{Z}$ and any prime p we consider $N_X(p)$ the number of $\mathbf{F}_p$-points of the scheme $X/\mathbf{F}_p$. Given a in $\mathbf{Z}$, we study the set $\{p : p \nmid N_X(p) - a\}$. In case $\dim X$ is small (lower than 3), we give a simple criterion for this set to be infinite and in this case we prove it has positive lower density.

ALEXANDER DUNCAN, University of South Carolina
Exceptional collections on arithmetic toric varieties

An "arithmetic toric variety" is a normal variety with a faithful action of an algebraic torus having a dense open orbit. When the base field is algebraically closed, there is only one torus in every dimension and one can identify the torus with its orbit. Over a general field, there may be many non-isomorphic tori of the same dimension. Moreover, it is no longer possible to identify the torus with its orbit since there may not exist any rational points.

Exceptional collections are one way of describing the bounded derived categories of coherent sheaves on a variety. The existence of exceptional collection is a very strong condition but, nevertheless, Kawamata showed that all smooth projective toric varieties possess exceptional collections when the ground field is algebraically closed. For a general field, this immediately fails even in dimension 1. However, if one allows an arithmetic generalization of the "usual" notion of exceptional object, then the theory is again interesting.

MATTHIAS FRANZ,
Symmetric products of maximal varieties

Let X be a complex algebraic variety equipped with an antiholomorphic involution τ. Then the mod 2 Betti sum of the real part X^τ cannot exceed the mod 2 Betti sum of X. In case of equality one calls X *maximal* or an *M-variety*. Biswas–D'Mello have shown that if a compact connected Riemann surface, say of genus g, is maximal, then so is its n-th symmetric power for $n \leq 3$ and $n \geq 2g-1$. We show that this holds for any n. As we will explain, this is actually a purely topological statement about symmetric products and, more generally, Γ-products of equivariantly formal spaces.

CHRISTIAN MAIRE, Université Bourgogne Franche-Comté, Cornell University
Unramified pro-p extensions of number fields

In this talk, I will focus on the structure of the Galois groups of unramified pro-p extensions of number fields: cohomological dimension, relations, etc. As application, I will give new records on liminf of root discriminants of number fields (Martinet's constant), and produce infinite unramified extensions where the set of splitting is infinite (Ihara's question). We will also show how such extensions appear naturally in the context of p-rational fields. Joint work with F. Hajir (UMass) and R. Ramakrishna (Cornell U.)

FEDERICO PASINI, University of Western Ontario
Koszul property in Galois cohomology

Absolute Galois groups of fields are a main object of interest in algebraic number theory and related subjects, but we are very far from a satisfactory understanding of their structure. Yet, recently a great breakthrough has been obtained with the proof of Bloch-Kato conjecture. This gave mathematicians the first substantial insight on the rather mysterious Galois cohomology of an absolute Galois group (and of its pro-p quotients), an important invariant of a field. Its most significant consequence is that, in case a field contains a primitive p-th root of unity, the Galois cohomology of its absolute Galois group with coefficients in $\mathbb{F}_p$ is a quadratic algebra. There is a class of quadratic algebras with an uncommonly good homological behaviour and endowed with a useful duality functor: the class of Koszul algebras. L. Positselski conjectured that in the above situation the Galois cohomology of absolute Galois groups is always Koszul, and proved this for various classes of fields, e.g. for algebraic number fields.

We discuss the meaning of Koszulity in the framework of Galois theory and we show some new ways to prove Koszulity of Galois cohomology for other significant classes of fields.

This is a joint work with Jan Minac, Marina Palaisti, Claudio Quadrelli, Tan Nguyen Duy.

ANDREW SCHULTZ, Wellesley College
Galois module structure of p^sth power classes of a field

When a field K contains a primitive pth root of unity, Kummer theory tells us that the $\mathbb{F}_p$-space $K^{\times p}/K^{\times}$ is a parameterizing space for elementary p-abelian extensions of K. In previous work, the authors computed the Galois module structure of this set when the Galois group came from an extension K/F whose Galois group is isomorphic to $\mathbb{Z}/p^n\mathbb{Z}$. In this talk we consider the more refined group $K^{\times p^s}/K^{\times}$ as a Galois module, and we report on progress in computing its structure. It appears that there is only one summand which is not free (either under the full ring or one of its natural quotients), and this summand's structure seems to be connected to the cyclotomic character and a certain family of embedding problems along the tower K/F. This work is joint with Ján Mináč and John Swallow.

ANNE SHEPLER, University of North Texas
Resolutions for twisted tensor products

Finding the cohomology of a module or algebra is often hampered by lack of a convenient explicit resolution. Yet many algebras are generated by smaller subalgebras with nice resolutions. We give a technique for weaving together resolutions of smaller algebras to create a resolution for a parent algebra in order to determine cohomological data. In particular, we consider algebras that arise as the product of two subalgebras as a vector space, introducing *twisted tensor resolutions* for twisted tensor products. Many noncommutative algebras manifest as twisted tensor products: Weyl algebras, quantum polynomial rings, Ore extensions in general, Koszul pairs, Sridharan algebras, smash products of groups acting on Koszul rings, semi-direct product algebras in general, and universal enveloping algebras of Lie algebras, for example. The Chevalley-Eilenberg resolution for computing Lie algebra cohomology arises as a special case of the twisted tensor resolution. This construction is also helpful for computing Hochschild cohomology and finding deformations. Joint with Sarah Witherspoon.

VAIDEHEE THATTE, Queen's University, Kingston, ON
Ramification Theory II: Refined Swan Conductor and Defect

Classical ramification theory deals with extensions of complete discrete valuation rings with perfect residue fields. We would like to study arbitrary valuation rings with possibly *imperfect* residue fields and possibly *non-discrete* valuations of rank ≥ 1, since several fascinating complications arise for such rings. In particular, defect may occur (i.e. we can have a non-trivial extension, such that there is no extension of the residue field or the value group) when the residue characteristic is positive.

In "Ramification Theory I", we presented a generalization of *Kato's Swan conductor* for degree p extensions of arbitrary valuation fields in residue characteristic $p > 0$. Now we discuss a generalization and further refinement of the *refined Swan conductor* in this case. Our hope is that these results will have many interesting applications in algebraic geometry and number theory.

CHANGLONG ZHONG, SUNY-Albany
Algebraic definition of K-theoretic stable bases

In this talk I will talk about the algebraic definition of K-theoretic stable bases for Springer resolutions, originally defined by Maulik and Okounkov. It is generated by the convolution action of the affine Hecke algebra. I will then mention the restriction formula. This is joint work with Changjian Su and Gufang Zhao.