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Some refinements of Artin's conjecture

In 1927, Artin gave a heuristic argument that 2 is a primitive root (mod p) approximately 37% of the time. No one has been able to make his argument rigorous, and even the weaker problem of showing that 2 is a primitive root (mod p) for infinitely many p remains open.

Artin's initial heuristic has been generalized, giving rise to conjectures on the proportion of primes p for which any given integer is a primitive root (mod p); the most general form of this is now known as Artin's conjecture. In this talk I will describe several new conjectures (joint with Greg Martin, UBC) on the proportion of the time a given integer is "almost" a primitive root (mod p). Our conjectures subsume Artin's conjecture, and are borne out in computations. I'll also prove that our conjectures hold on average, and derive some consequences of this. For example, we obtain a new proof that Artin's conjecture holds on average, a result originally due to Goldfeld.