Geometric Potential Theory
Théorie du potentiel en géométrie
(Org: **Jie Xiao** (Memorial University))

GRAHAM COX, Memorial University
A dynamical approach to semilinear elliptic equations

In this talk I will describe a new procedure for reducing a semilinear elliptic PDE on a bounded domain $\Omega \subset \mathbb{R}^n$ to an infinite-dimensional dynamical system on the boundary $\partial\Omega$.

Suppose u satisfies the equation $\Delta u + F(x,u)$ on Ω. When the domain is deformed through a one-parameter family $\{\Omega_t\}$, the Cauchy data of u on $\partial\Omega_t$ will satisfy a first-order evolution equation. This equation is ill-posed, in the sense that it does not admit solutions forwards or backward in time for generic initial data. However, if the domain is deformed smoothly to a point, this equation admits an exponential dichotomy, so there exist two distinguished subspaces of boundary data (at each time t) for which solutions exists forward and backward in time, respectively. When the PDE is selfadjoint, the evolution equation will be Hamiltonian, so the unstable subspace is Lagrangian and hence has a well defined Maslov index. These constructions generalize previous work in spatial dynamics, which considered elliptic equations on cylindrical domains.

This is joint with with Margaret Beck, Christopher Jones, Yuri Latushkin and Alim Sukhtayev.

QINGSONG GU, Memorial University
Dirichlet forms and critical exponents on fractals

Let $B^{\sigma}_{2,\infty}$ denote the Besov space defined on a compact set $K \subset \mathbb{R}^d$ which is equipped with an α-regular measure μ. The critical exponent σ^* is the supremum of the σ such that $B^{\sigma}_{2,\infty} \cap C(K)$ is dense in $C(K)$. It is well-known that for many standard self-similar sets K, $B^{\sigma^*}_{2,\infty}$ are the domain of some local regular Dirichlet forms. In this talk, I will explain a new situation we have explored that the underlying fractal sets admit inhomogeneous resistance scalings, which yield two types of critical exponents. We developed a general theory of this on the p.c.f. sets. Our emphasis is on two asymmetric p.c.f. sets that are constructed. We use them to illustrate and examine the theory, the function properties of the associated Besov spaces at the critical exponents, and also the Dirichlet forms on these fractals. This is a joint work with Ka-Sing Lau.

SALVATORE LEONARDI, Department of Mathematics and Informatics, University of Catania, Italy
Regularity results for solutions to some classes of nonlinear elliptic equations

We deal with the regularity of a solution of the Dirichlet problem associated to the singular equation

$$-\mathrm{div}(a(x)Du) + M\,\frac{|Du|^2}{u^\theta} = f(x) \ \text{ in } \Omega \tag{1}$$

where Ω is an open bounded subset of $\mathbb{R}^N$ $(N \geq 3)$ with smooth boundary, $a(x)$ is a L^∞–matrix satisfying the standard ellipticity condition, $\theta \in]0,1[$, M is a positive constant and f is sufficiently regular i.e. it belongs to a suitable Morrey space. Namely, we assume that the right-hand side f belong to the Morrey space $L^{m,\lambda}(\Omega)$ with

$$1 \leq m \leq \frac{2N}{2N - \theta(N-2)} \quad \text{and} \quad 0 < \lambda < N-2$$

so that our right-hand side doesn't belong to the natural dual space or it is "nearly" a measure $(m = 1)$.

We will be concerned with the regularity of the gradient of a solution in Morrey spaces in correspondence with the Morrey properties of the right–hand side of the equation (1).

LIGUANG LIU, Renmin University of China
Gaussian Capacity Theory

In this talk, we will discuss the Sobolev-capacity theory in Gauss space and applications to tracing the Gaussian Sobolev-space as well as the induced geometric structure.

NICO LOMBARDI, Memorial University of Newfoundland and University of Firenze
Fractional Sobolev trace inequalities

We will present some Sobolev type inequalities regarding the trace of a function for the half-space $\mathbb{R}^n_+$ in the classical and fractional case.

We will start to show the result due to Escobar and independently to Beckner: there exists a positive constant K_n, depending only on n, such that for any function $f \in W^{1,2}(\mathbb{R}^n_+)$, it holds

$$(\int_{\mathbb{R}^{n-1}} |f(0,x)|^{\frac{2(n-1)}{n-2}} dx)^{\frac{n-1}{n-2}} \leq \ K_n \int_{\mathbb{R}^n_+} |\nabla f(t,x)|^2 dxdt,$$

where $(t,x) \in \mathbb{R}_+ \times \mathbb{R}^{n-1} = \mathbb{R}^n_+$.

Afterwards we will consider the case of fractional Sobolev inequalities, presenting the fractional counterpart of the previous statement and some possible generalizations. (This is a work in progress with Jie Xiao)

JAVAD MASHREGHI, Laval University
Zero sets and capacity

While the Blaschke condition completely settles the question of zero sets and uniqueness sets in the Hardy space H^p, the corresponding question is yet open in several closely related function spaces. In the classical Dirichlet space, there is a sufficient condition by H. Shapiro, Shields for the zeros sets, whose sharpness is crystalized by a result of Nagel, Rudin, J. Shapiro. A complete characterization in not available yet. We provide some new results which are not covered by the classical theorems.

Joint work with K. Kellay.

EVAN MILLER, University of Toronto
Enstrophy Growth and the Navier Stokes Strain Equation

In this talk I will derive an evolution equation for the symmetric part of the gradient (the strain tensor) in the incompressible Navier Stokes equation, and prove the existence of L^2 mild solutions for this equation locally in time. I will use this PDE to derive a simplified identity for the growth of enstrophy for mild solutions that depends only on the strain tensor, not on the interaction of the strain tensor with the vorticity; this will also allow a substantial improvement of the constant in the differential inequality for enstrophy growth of the form $\partial_t E(t) \leq CE(t)^3$. I will use this to prove a lower bound on blow-up time in terms of the initial enstrophy, as well as provide analytical evidence for the observed alignment of vorticity to the middle eigenvector of the strain matrix. Finally, I will consider the variational problem related to enstrophy growth that corresponds to maximizing the instantaneous rate of enstrophy growth.

YUHUA SUN, Nankai University
On nonexistence and existence of positive solutions to semilinear elliptic and parabolic problems on manifolds

We determine the critical exponent for certain semi-linear elliptic problem on Riemannian manifolds assuming the volume regularity and Green function estimates. By using a sharp volume condition, we also reinvestigate nonexistence and existence of global positive solutions to semilinear parabolic equation on Riemannian manifolds. This talk is based on joint works with Prof. Grigor'yan, and Fanheng Xu.

XIAOMIN TANG, Huzhou University
Schwarz lemma at the boundary and rigidity property for holomorphic mappings on the unit ball of $\mathbb{C}^n$

Schwarz lemma at the boundary and rigidity property for holomorphic mappings on the unit ball of $\mathbb{C}^n$

Xiaomin Tang

Huzhou University

In this talk, we first establish a new type of the classical Schwarz lemma at the boundary for holomorphic self-mappings of the unit ball in $\mathbb{C}^n$, and then give the boundary version of the rigidity theorem. This is a joint work with Taishun Liu and Wenjun Zhang.

JIANFEI WANG, Zhejiang Normal University
The Roper-Suffridge extension operator and its applications to convex mappings

The talk is twofold. The first is to investigate the Roper-Suffridge extension operator which maps a biholomorhic function f on D to a biholomorphic mapping F on

$$\Omega_{n,p_2,\cdots,p_n}(D) = \left\{ (z_1, z_0) \in D \times \mathbb{C}^{n-1} : \sum_{j=2}^{n} |z_j|^{p_j} < \frac{1}{\lambda_D(z_1)} \right\}, \ \ p_j \geq 1,$$

where $z_0 = (z_2, \ldots, z_n)$ and λ_D is the density of the Poincar$\acute{e}$ metric on a simply connected domain $D \subset \mathbb{C}$. We prove this Roper-Suffridge extension operator preserves ε-starlike mapping: if f is an ε-starlike, then so is F. As a consequence, we solve a problem of Graham and Kohr in a new method. By introducing the scaling method, the second part is to construct some new convex mappings of domain $\Omega_{2,\,m} = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^m < 1\}$ with $m \geq 2$, which can be applied to discuss the extremal point of convex mappings on the domain. This scaling idea can be viewed as providing an alternative approach to study convex mappings on $\Omega_{2,\,m}$. Moreover, we propose some problems.

SUDAN XING, Memorial University
The general dual Orlicz-Minkowski problem

The classical Minkowski problem is a central problem in convex geometry which asks that given a nonzero finite Borel measure μ, what are the necessary and sufficient conditions on μ such that μ equals to the surface area measure of a convex body K.

My presentation is about the general dual extension of the classical Minkowski problem—the general dual Orlicz-Minkowski problem. That is, *for which nonzero finite Borel measures μ on S^{n-1} and continuous functions G and ψ do there exist a constant $\tau \in \mathbb{R}$ and a convex body K such that $\mu = \tau\, \widetilde{C}_{G,\psi}(K,\cdot)$?* Here $\widetilde{C}_{G,\psi}(K,\cdot)$ is the finite signed Borel measure. In particular, a solution to this problem will be presented. This talk is based on a joint work with Richard Gardner, Daniel Hug, Wolfgang Weil and Deping Ye.

DACHUN YANG, School of Mathematical Sciences, Beijing Normal University
Multiplication Between Hardy Spaces and Their Dual Spaces

It is well known that bilinear decompositions of products of Hardy spaces and their dual spaces play an important role in the study on various problems from analysis. In this talk, we present some recent progresses on such bilinear decompositions of products of Hardy spaces and their dual spaces. Some open questions are also mentioned in this talk.

DEPING YE, Memorial University of Newfoundland
The Orlicz-Petty bodies

The geominimal surface areas are fundamental objects in convex geometry, which are continuous and affine invariant. In this talk, I will discuss how the homogeneous and nonhomogeneous Orlicz geominimal surface areas are defined via the optimization problems related to the Orlicz mixed volumes. In particular, I will present, under certain conditions, the existence and continuity of Orlicz-Petty bodies. This talk is based on the joint work with Zhu and Hong (Int. Math. Res. Notices, 2018)

WEN YUAN, Beijing Normal University
Functional Calculus on BMO-type Spaces of Bourgain, Brezis and Mironescu

A nonlinear superposition operator T_g related to a Borel measurable function $g:\ \mathbb{C} \to \mathbb{C}$ is defined via $T_g(f) := g \circ f$ for any complex-valued function f on $\mathbb{R}^n$. In this talk, we investigate the mapping properties of T_g on a new BMO type space recently introduced by Bourgain, Brezis and Mironescu [J. Eur. Math. Soc. (JEMS) 17 (2015), 2083-2101], as well as its VMO and CMO type subspaces. Some sufficient and necessary conditions for the inclusion result and the continuity property of T_g on these spaces are obtained.

KELVIN ZHANG, University of Toronto
On concavity of the principal's profit maximization facing agents who respond nonliearly to prices.

A monopolist wishes to maximize her profits by finding an optimal price menu. After she announces a menu of products and prices, each agent will choose to buy that product which maximizes his own utility, if positive. The principal's profits are the sum of the net earnings produced by each product sold. These are determined by the costs of production and the distribution of products sold, which in turn are based on the distribution of anonymous agents and the choices they make in response to the principal's price menu. In this talk, we describe a necessary and sufficient condition for the convexity or concavity of the principal's problem, assuming each agent's disutility is a strictly increasing but not necessarily affine (i.e. quasilinear) function of the price paid. Concavity when present, makes the problem more amenable to computational and theoretical analysis; it is key to obtaining uniqueness and stability results for the principal's strategy in particular. Moreover, we present the analytic and geometric interpretations of this condition. Even in the quasilinear case, our analysis goes beyond previous work by addressing convexity as well as concavity, by establishing conditions which are not only sufficient but necessary, and by requiring fewer hypotheses on the agents' preferences. This talk represents joint work with my Ph.D. advisor Robert McCann.

NING ZHANG, Kent State University
A solution to the problem of bodies with congruent sections or projections

In this talk, we will construct two convex bodies K and L, such that their projections $K|H$, $L|H$ onto every subspace H are congruent, but nevertheless, K and L do not coincide up to a translation or a reflection in the origin. This gives a negative answer to an old conjecture posed by Nakajima and Süss.