Categories and Topology
Catégories et topologie

(Org: **Darien DeWolf** (St. Francis Xavier University), **Pieter Hofstra** (University of Ottawa / Université d'Ottawa) and/et **Dorette Pronk** (Dalhousie University))

EVANGELIA ALEIFERI, Dalhousie University
Spans as Cartesian Double Categories

In this talk, we give a characterization of the double category of Spans as a Cartesian double category. We say that a double category $\mathbb{D}$ is Cartesian if the diagonal double functor $\Delta : \mathbb{D} \to \mathbb{D} \times \mathbb{D}$, and the unique double functor $! : \mathbb{D} \to 1$, have right adjoints. This work was motivated by a characterization of the bicategory of Spans, that was given in *Bicategories of spans as cartesian bicategories* by Lack, Walters, and Wood.

MARZIEH BAYEH, Dalhousie University
Higher Equivariant Topological Complexity

Topological complexity was introduced by Farber in 2003 to estimate the complexity of a motion planning algorithm. Topological complexity of a space X, denoted by $TC(X)$, is the least number of open sets that form a covering for $X \times X$ in which each open set admits a section to the endpoints map $\pi : PX \to X \times X$, where $PX = X^I$ is the path space of X.

In 2010, Rudyak introduced a series of invariants $\{TC_n(X)\}_n$ and called them higher topological complexity. These invariants can be considered as generalizations of the topological complexity.

In this talk, we study an equivariant version of the higher topological complexity for a topological space which admits an action of a topological group. We consider the diagonal action on $X \times X$ and show some properties of the higher equivariant topological complexity.

MATT BURKE, University of Calgary
The Calculus of Infinity Functors and Tangent Categories

In classical calculus we approximate an appropriately differentiable function using a sequence of simpler functions called the Taylor polynomials. In an analogous way the functor calculus describes how to approximate a functor whose domain and codomain are appropriately topological by using a sequence of simpler functors. Since its introduction by Goodwillie several different variants of the functor calculus have been studied. A first step towards the unification of some these variants was made in the paper [1] which constructs a directional derivative in the calculus of functors that satisfies all the axioms for being a Cartesian differential category. Cartesian differential categories [2] provide an axiomatisation of the salient features of differentiable functions between Euclidean spaces.

In this talk we describe an ongoing project to formulate the fundamental parts of the Goodwillie calculus in terms of a tangent structure on the category of presentable infinity categories. A tangent structure on a category is an axiomatisation of the tangent bundle functor on the category of smooth manifolds that was introduced by Rosicky in 1984 and further extended in [3]. This work is part of a joint project with Kristine Bauer and Michael Ching.

[1] Bauer, Johnson, Osborne, Riehl, Tebbe. Directional derivatives and higher order chain rules for abelian functor calculus. Topology Appl. 235 (2018), 375–427.

[2] Blute, Cockett, Seely. Cartesian differential categories. Theory Appl. Categ. 22 (2009), 622–672.

[3] Cockett, Cruttwell. Differential structure, tangent structure, and SDG. Appl. Categ. Structures 22 (2014), no. 2, 331–417.

ROBIN COCKETT, University of Calgary

Constructions and Connections in Tangent Categories

The attraction of a categorical doctrine (such as that of tangent categories) is enhanced if it permits natural constructions. We shall look at constructions using connections and show that, indeed, they generate (new) tangent categories.

Time permitting we shall explore the form that classical mechanics takes in a tangent category (this is work with Ben MacAdam).

DARIEN DEWOLF, St. Francis Xavier University

Groupoids Associated to Join Inverse Categories

Join inverse categories [1] are inverse categories that come with a way to glue together two partially defined morphisms, provided they agree everywhere that they are defined. Every inverse semigroup can be associated to an inductive groupoid (Ehresmann-Schein-Nambooripad), and by extension every inverse category $\mathbf{X}$ to a top-heavy locally inductive groupoid $\mathcal{G}(\mathbf{X})$ [2].

This talk will show that the groupoids associated to join inverse categories share at least two interesting properties [3]:

(1) Each admits a pair of functors

$$(-)_* : \mathcal{G}(\mathbf{X})^{\mathrm{op}} \to \mathbf{Loc} \text{ and } (-)^* : \mathcal{G}(\mathbf{X}) \to \mathbf{Loc}$$

such that, for each arrow $(\alpha : A \to B) \in \mathcal{G}(\mathbf{X})$, the locale homomorphisms α_* and α^* form an equivalence of categories between A^* and B^*.

(2) Each admits an Ehresmann topology, a data structure very much analogous to a Grothendieck topology, but in the language of *covering order ideals*.

References

[1] J.R.B. Cockett, G.S.H. Cruttwell, and J.D. Gallagher. Differential restriction categories. Theory and Applications of Categories, 25(21):537-613, 2011.

[2] D. DeWolf and D. Pronk. The Ehresmann-Schein-Nambooripad Theorem for Inverse Categories. arXiv, Nov. 2017, 1507.08615v2.

[3] D. DeWolf. Restriction Category Perspectives of Partial Computation and Geometry. PhD thesis, Dalhousie University, 2017.

JONATHAN GALLAGHER, University of Calgary

Weil algebras and Smootheology

There is no known category of manifolds that is cartesian closed, yet there have been various generalizations of manifolds that are: diffeological spaces (Souriau), differential spaces (Sikorksi), Frölicher spaces, and Weil spaces (Bertram, Dubuc, Nishimura).

In this talk, we will explore these categories of generalized smooth spaces from the point of view of Weil algebra actions. Leung's theorem says that to have a tangent structure is to have a Weil-actegory structure that interacts well with transverse limits. We will then show how to extract a cartesian closed tangent category from each of these settings together with a coherence that allows two features: the categories admit a model of the differential lambda-calculus of Ehrhard-Regnier, and the categories have a cartesian closed embedding into a representable tangent category, extending a result of Garner.

NICK GURSKI, Case Western Reserve University
Sign conventions, higher supergeometry, and the two-type of the sphere

In his recent exposition of supergeometry, Kapranov outlines a philosophy that sign conventions in higher supergeometry should arise from categorical models for the truncated sphere spectrum. A model for the 1-type of the sphere is relatively easy to construct, while the 2-type has been far more elusive. I will explain why everyone's first guess for such an object cannot work, and go on to explain how a model using chain complexes of Picard categories does. This model uses a higher categorical analogue of homological algebra, and in fact demonstrates a certain universality to Kapranov's philosophy. This is joint work with Niles Johnson, Angélica Osorno, and Marc Stephan.

BRENDA JOHNSON, Union College
Functor Precalculus

Functor calculi have been developed in a variety of forms and contexts in algebra and topology. Each of these calculi comes equipped with its own definition of polynomial or degree n functor. Such definitions are often formulated in terms of the behavior of the functor on certain types of cubical diagrams. Using the discrete calculus developed with Kristine Bauer and Randy McCarthy as a starting point, we identify a category-theoretic framework, which we call a precalculus, that provides a means by which notions of degree for functors can be defined via cubical diagrams. We show how such precalculi might be used to produce functor calculi. This is work in progress with Kathryn Hess.

CHRIS KAPULKIN, University of Western Ontario
Cubical sets and the homotopy coherent nerve

Cubical sets are a well-studied model for the homotopy theory of spaces, providing in many cases a convenient alternative to simplicial sets. However, while there is only one category simplicial sets, there are multiple categories of cubical sets, depending on the choice of morphisms in the indexing category $\square$.

For a suitable choice of the category $\square$, we construct a topology on it, the sheaves for which are precisely simplicial sets. That gives a full embedding of the category of simplicial sets into the category of cubical sets. We further generalize several constructions of higher category theory from simplicial sets to cubical sets, including the homotopy coherent nerve and Lurie's straightening-unstraightening construction.

This talk is based on joint work with Vladimir Voevodsky "Cubical approach to straightening".

MICHAEL LAMBERT, Dalhousie University
A Tensor Product of Fibrations as a Codescent Object

This talk will give an explicit construction of a tensor product of an opfibration and a fibration over the same base category. That this is indeed a tensor product is attested to by an associated tensor-hom adjunction; and by the fact that tensoring with a fixed opfibration induces a category-valued 2-functor that, under conditions generalizing the well-known characterization of flat set-valued functors in terms of filtered categories, is left exact. Ordinary fibrations and opfibrations are known to be algebras for certain colax-idempotent 2-monads. The explicit tensor product construction can be used to show that it is a codescent object of coherence data arising from the structure maps coming with the opfibration and fibration.

JS LEMAY, University of Oxford
Differential Categories and Representable Tangent Categories

Differential categories were introduced to provide categorical models of differential linear logic and in particular come equipped with a natural transformation d, called the *deriving transformation*, whose axioms are based on basic properties of the derivative. CoKleisli categories of differential categories are very well studied as they provide models of the differential λ-calculus, but co-Eilenberg-Moore categories of differential categories are not as well studied. Tangent categories are categories which come equipped with an endofunctor T whose axioms capture the basic properties of the tangent bundle of a smooth manifold. The link between synthetic differential geometry and tangent categories is captured by the notion of a representable tangent category, which is a tangent category such that $\mathsf{T} = (-)^{\mathsf{D}}$ for some object D known as an infinitesimal object. In this talk we explain how a co-Eilenberg-Moore category of a differential category with sufficient equalizers is a representable tangent category, and also how every Eilenberg-Moore category of a codifferential category is a tangent category.

RORY LUCYSHYN-WRIGHT, Mount Allison University
Representation of probability measures and compacta through dualization of convex spaces and Sierpinski modules

The concept of dualization of linear spaces admits a far-reaching generalization through the notion of an *algebraic duality* [3], i.e., a contravariant adjunction between (enriched) algebraic categories. Every algebraic duality is induced by a *dualizing algebra* and determines an induced notion of *distribution* that specializes to yield various kinds of measures, Schwartz distributions, filters, closed subsets, compacta, and so forth [2].

In this talk, we will consider three examples of algebraic dualities enriched in the category of convergence spaces, one involving convex spaces and *based involutive convex spaces*, and two involving modules for the *Sierpinski rig* or the *dual Sierpinski rig*. We show that for locally compact Hausdorff spaces the induced notions of distribution in these examples are the notions of Radon probability measure, closed subset, and compact subset, respectively. In proving these results we employ R. C. Buck's representation theorem for bounded measures [1] as well as the Hofmann-Mislove theorem.

[1] R. C. Buck, Bounded continuous functions on a locally compact space. *The Michigan Mathematical Journal* 5 (1958) 95—104.

[2] R. B. B. Lucyshyn-Wright, Functional distribution monads in functional-analytic contexts. *Advances in Mathematics* 322 (2017), 806–860.

[3] R. B. B. Lucyshyn-Wright, Algebraic duality and the abstract functional analysis of distribution monads. Talk at *CT 2017: International Category Theory Conference*, Vancouver, July 2017.

NICHOLAS MEADOWS,
Local Higher Category Theory

We describe local presheaf theoretic extensions of three of the main extant models of higher category theory: the Joyal, Bergner and Rezk model structures, in which the weak equivalences are defined 'stalkwise'. There is a zig-zag of Quillen equivalences between them, which extends the Quillen equivalences linking the various models of higher category theory.

The local Bergner model structure is right proper. This leads to an attractive theory of cocycles and torsors, which generalizes classical non-abelian H^1.

The local Joyal model structure leads to new perspectives on the theory of higher stacks (following Simpson), which will be briefly discussed.

SIMONA PAOLI, University of Leicester
Comonad cohomology of track categories

Simplicial categories are one of the models of $(\infty, 1)$-categories. They can be studied using the Postnikov decomposition, whose sections are categories enriched in simplicial n-types and whose k-invariants are defined in terms of the (S,O)-cohomology of Dwyer, Kan and Smith. The latter is defined topologically, while the understanding of the k-invariants calls for an algebraic description. In this talk I illustrate the first step of this program, for categories enriched in groupoids, also called track category. We define a comonad cohomology of track categories and we show that, under mild hypothesis on the track category, its comonad cohomology coincides up to a dimension shift with its (S,O)-cohomology, therefore obtaining an algebraic formulation of the latter. This is joint work with David Blanc.

JUAN PABLO QUIJANO, IST - University of Lisbon
Functoriality of étale groupoids (joint work with Pedro Resende)

The correspondence between étale groupoids and inverse quantal frames extends naturally to a module theoretic description of actions and sheaves for étale groupoids whereby, for instance, sheaves can be identified with Hilbert quantale modules equipped with a Hilbert basis. The latter correspondence is functorially well behaved, but the correspondence between étale groupoids and inverse quantal frames themselves is not functorial with respect to the usual notions of morphism (groupoid functors and homomomorphisms of unital involutive quantales). Instead, it is functorial in a bicategorical sense: the bicategory $\mathbf{Gpd}$, of localic étale groupoids with bi-actions as 1-cells, is bi-equivalent to the bicategory $\mathbf{IQLoc}$, whose objects are the inverse quantal frames and whose 1-cells are quantale bimodules satisfying a mild condition.

Taking advantage of these facts we shall describe principal bundles, Hilsum–Skandalis maps, and Morita equivalence in terms of modules on inverse quantal frames. The Hilbert module description of quantale sheaves leads naturally to a formulation of Morita equivalence in terms of bimodules that resemble imprimitivity bimodules of C*-algebras.

References:

[1] P. Resende, Étale groupoids and their quantales, *Adv. Math.* 1 (2007) 147–209.

[2] P. Resende, Groupoid sheaves as quantales sheaves, *J. Pure Appl. Algebra* 216 (2012) 41–70.

[3] P. Resende, Functoriality of groupoid quantales. I, *J. Pure Appl. Algebra* 219 (2015) 3089–3109.

[4] J. Quijano and P. Resende, Functoriality of groupoid quantales. II, *arXiv:1803.01075* (2018).

EGBERT RIJKE, Carnegie Mellon University
The join construction

In homotopy type theory we can define the fiberwise join of maps as a binary operation on maps with a common codomain, by first taking the homotopy pullback, and then the homotopy pushout. This operation is commutative, associative, and the unique map from the empty type into the common codomain is a neutral element. Moreover, the idempotents of the join of maps are precisely the embeddings.

We define the image of a map $f : A \to X$ as the colimit of the finite join powers of f. This construction of the image is called the join construction. The join powers therefore provide approximations of the image inclusion. These approximations can be of interest themselves: for instance the projective spaces appear as such.

Furthermore, we show that if A is essentially small (with respect to a universe $\mathcal{U}$), and X is locally small (w.r.t. $\mathcal{U}$), then the image of f is again essentially small. This observation helps us to obtain a type theoretic version of the replacement axiom, and we use it to show that quotients of small types are again small.

Using the idea of (higher) quotients we are then able to show that for any reflective subuniverse (a subuniverse of which the inclusion functor has a left adjoint), the subuniverse of separated objects (of which the identity type is in the original reflective subuniverse) is again a reflective subuniverse.

LAURA SCULL, Fort Lewis College
Equivariant fundamental groupoids as categorical constructions

The equivariant fundamental groupoid, first defined by tom Dieck, is a category which incorporates the fundamental groupoids of all of the fixed sets of a G-space X. It was later reinterpreted as a Grothendieck construction of the fundamental groupoid functor. When considering compact Lie groups with their own topology, tom Dieck also defined a discrete version of this category, equating certain homotopies coming from within the group structure. This can also be interpreted using a Grothendieck construction, but it needs to be considered as a 2-functor into a 2-category.

My goal in this talk is to introduce the various fundamental groupoids, with examples, and show how they follow naturally from a study of equivariant topology. I will then explain how to view them categorically, illustrating how these higher categorical structures capture the topology. As time allows, I will discuss how this interpretation can be used to show a Morita equivalence for representable groupoids.

This is joint work with D. Pronk at Dalhousie University.

SARAH YEAKEL, University of Maryland
Chain rules and operads in abelian functor calculus

The abelian functor calculus of Johnson and McCarthy associates to a functor of abelian categories a sequence of "polynomial" approximations analogous to a Taylor series. Work of Bauer, Johnson, Osborne, Riehl, and Tebbe shows that in this setting, a directional derivative can be defined which yields a higher order chain rule. In ongoing work with Bauer and Johnson, we have found another chain rule, this time for the higher order differential. Using this chain rule, we define an operad structure on the derivatives of monads of R-modules, and more generally, find a monoidal structure for the derivatives of functors of abelian categories. We expect this to lead to classifications of homogeneous and polynomial functors of abelian categories.