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Functoriality of étale groupoids (joint work with Pedro Resende)

The correspondence between étale groupoids and inverse quantal frames extends naturally to a module theoretic description of actions and sheaves for étale groupoids whereby, for instance, sheaves can be identified with Hilbert quantale modules equipped with a Hilbert basis. The latter correspondence is functorially well behaved, but the correspondence between étale groupoids and inverse quantal frames themselves is not functorial with respect to the usual notions of morphism (groupoid functors and homomomorphisms of unital involutive quantales). Instead, it is functorial in a bicategorical sense: the bicategory \mathbf{Gpd} , of localic étale groupoids with bi-actions as 1-cells, is bi-equivalent to the bicategory \mathbf{IQLoc} , whose objects are the inverse quantal frames and whose 1-cells are quantale bimodules satisfying a mild condition.

Taking advantage of these facts we shall describe principal bundles, Hilsum–Skandalis maps, and Morita equivalence in terms of modules on inverse quantal frames. The Hilbert module description of quantale sheaves leads naturally to a formulation of Morita equivalence in terms of bimodules that resemble imprimitivity bimodules of C^* -algebras.

References:

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