RENATE SCHEIDLER, University of Calgary
Dan Shanks' CUFFQI Algorithm Resurrected

In 1925, William E. H. Berwick designed an approach for enumerating all cubic fields $\mathbb{K}$ of a given fixed discriminant Δ via suitable integers, which he termed "quadratic generators", in the quadratic resolvent field $\mathbb{Q}(\sqrt{-3\Delta})$ of $\mathbb{K}$. When Δ is fundamental, he showed in particular that every cubic field $\mathbb{K}$ of discriminant Δ has a generating polynomial of the form $f_\lambda(x) = x^3 - 3(\lambda\overline{\lambda})^{1/3}x + (\lambda + \overline{\lambda}) \in \mathbb{Z}[x]$ where $(\lambda) = \mathfrak{a}^3$ and $\mathfrak{a}$ is an ideal in the maximal order of $\mathbb{Q}(\sqrt{-3\Delta})$.

Unfortunately, the Berwick construction can produce generating polynomials with very large coefficients. For example, if $\Delta < 0$ and λ is the fundamental unit of $\mathbb{Q}(\sqrt{-3\Delta})$, then $f_\lambda(x) = x^3 \pm 3x + T$ where $T \approx \exp(\sqrt{-3\Delta})$. In 1987, Daniel Shanks devised an ingenious algorithm for finding quadratic generators λ whose norm and trace in $\mathbb{Q}(\sqrt{-3\Delta})$ are both small, utilizing the infrastructure of $\mathbb{Q}(\sqrt{-3\Delta})$ when $\Delta < 0$. Shanks called his method "CUbic Fields From Quadratic Infrastructure, or CUFFQI (pronounced "cuff-key") for short. Although implemented in 1990 by Gilbert Fung as part of his PhD thesis, the CUFFQI algorithm was never published. In this talk, we present a modern version of this algorithm.