Studc Student Research Session
Session scientifique pour étudiants du Comité étudiant
(Org: **Aram Demenjian** (UQAM) and/et **Jean Lagacé** (University of Montreal / Université de Montréal))

YVONNE ALAMA, McGill
On the existence of a periodic solution to a nonlinear ODE using the radii polynomial approach.

We will give a rigorous computer assisted proof to solve part of a conjecture from Galaktionov and Svirshchevskii on the existence of a periodic solution to a fourth order nonlinear ODE. This ODE is used in the study of flame extinction phenomena in turbulent flows. Our procedure will be to reduce the problem of finding a periodic solution to one of finding a root of a function in finite dimensions. We will then introduce the tools necessary to rigorously prove the existence of a root of that function using the radii polynomial approach in finite dimensions.

ALEXIS LANGLOIS-RÉMILLARD, Université de Montréal
Bratteli and the morphisms of boundary seam algebras

Temperley-Lieb algebras $\mathsf{TL}_n(q+q^{-1})$ are a well-known family of associative complex algebras introduced in 1971 in a mathematical physics context and studied with various ways. Two main tools of the studies of representations of those algebras are the diagrammatic definition and the cellular structure in the sense of Graham-Lehrer. One can alter and generalize the setting of Temperley-Lieb algebras to give birth to other structures that permit the study of different physical problems. In this talk, we will discuss the boundary seam algebras $\mathsf{B}_{n,k}(q+q^{-1})$, a generalization by means of special idempotents introduced in 2015, by using the cellular approach. The structure of the generic q case will be shown and the first steps toward a systematic study of the critical q case will be presented.

JASON PARKER, University of Ottawa
Isotropy Groups of Algebraic Theories

Every first-order geometric theory $\mathbb{T}$ has a classifying Grothendieck topos $\mathbb{C}$, which contains a canonical group object called its 'isotropy group', which we may call the isotropy group of the theory $\mathbb{T}$. In this talk I will present several new results about the isotropy groups of equational algebraic theories. First, I will explain how we can use the initial purely categorical definition of the isotropy group to obtain a more concrete syntactical description of the isotropy group of an equational algebraic theory. I will then illustrate how to compute the isotropy groups of several popular algebraic theories, including the theories of (commutative) monoids, (abelian) groups, and (commutative) unital rings. Finally, I will explain how the isotropy group is an invariant of algebraic theories that captures a notion of 'inner automorphism', which generalizes the familiar notion from the theories of monoids, groups, and rings.

MARTIN PHAM, University of Waterloo
Recurrent iterated function systems and chaos game representation (and cannabis)

Iterated function systems (IFS) are a method of generating fractals by constructing an operator on an appropriate metric space whose attractor is self-similar. The operator represents the collective action of a set of affine transformations and fractals are generated by recursively applying the operator towards a fixed point. Recurrent iterated function systems (RIFS) generalize IFS by introducing a digraph that prescribes which kinds of recursive combinations of affine transformations are permissible. Chaos game representation (CGR) is a method of visualizing one dimensional sequences, first introduced in the context of DNA sequences. CGR of DNA sequences have been shown to produce images with self-similar patterns. Presented is a discussion of the relationship between RIFS attractors and CGR via an application to samples of cannabis sativa from different regions around the world. A wavelet multiresolution analysis of both demonstrates scale-invariant patterns.

JAMES RICKARDS, McGill

Number theoretic intersection numbers on Riemann surfaces

Consider a Riemann surface R, which is given as the quotient of the hyperbolic upper half plane $\mathcal{H}$ by G, a discrete subgroup of $\mathsf{PSL}(2,\mathbb{R})$. A classical construction of closed geodesics on R comes from taking the (real) fixed points of a hyperbolic matrix in G, and forming the hyperbolic geodesic between them. We ask the question: "given two such geodesics, how many times do they intersect on R?" We will focus on the case of $G = \mathsf{PSL}(2,\mathbb{Z})$, in which these geodesics correspond to indefinite binary quadratic forms. We will also touch upon the case where R is a Shimura curve; this case relates to the work on explicit class field theory for real quadratic number fields by Darmon and Vonk.

YURIJ SALMANIW, McMaster University

Existence and Regularity of Solutions to Some Singular Parabolic Systems

Recently, authors have studied the existence and regularity of solutions to systems of partial differential equations featuring singular nonlinearities. Under homogeneous Dirichlet boundary conditions, these reaction terms may grow singular as one approaches the boundary. Equations of this form have applications in biology and physics, specifically in the study of enzyme kinetics and the study of the gravitational potential of self gravitating, spherically symmetric fluid. This talk will highlight some of the tools used to prove the existence and regularity of solutions to

$$\begin{cases} u_t = d\Delta u + \frac{f(x)}{u^p v^q}, \\ v_t = D\Delta v + \frac{g(x)}{u^r v^s}, \end{cases}$$

and

$$\begin{cases} u_t = d\Delta u + \frac{f_1(x)}{u^p} + \frac{f_2(x)}{v^q}, \\ v_t = D\Delta v + \frac{f_3(x)}{u^r} + \frac{f_4(x)}{v^s}. \end{cases}$$

After perturbing the system, a functional method is used to obtain uniform $L^k(\Omega)$ bounds for the regularized solution $(u_\varepsilon, v_\varepsilon)$. These bounds allow us to prove the existence of a positive, weak solution (u, v), and in some cases the solution can be shown to be classical. Further, the solution is shown to exist for all time, and in some cases remains bounded for all time. These results are generalizations of previous work done with Dr. Chen and Dr. Xu featuring similar nonlinearities.

RASOUL SHAHSAVARIFAR, University of New Brunswick

Approximation of Data Depth Revisited

Data depth is a measure of centrality of $q \in \mathbb{R}^d$ with respect to a data set $S \subset \mathbb{R}^d$. Among various notions, two depth functions halfspace depth (Tukey, 1975) and β-skeleton depth (Yang, 2017) are considered in this study. The halfspace depth of $q \in \mathbb{R}^d$ with respect to $S \subset \mathbb{R}^d$ is the minimum portion of the elements of S which are located in one side of a halfspace passing through q. For $\beta \geq 1$, the β-skeleton depth of q with respect to S is the total number of β-skeleton influence regions that contain q, where each influence region is the intersection of two hyperballs obtained from a pair of points in S. Due to the hardness of computing the depth functions in some cases, approximation of depth functions is of interest. In this study, different methods are presented to approximate the halfspace and β-skeleton depth. First, an approximation technique is proposed to approximate the halfspace depth using the β-skeleton depth. Two dissimilarity measures based on the concepts of *fitting function* and *Hamming distance* are defined to train the halfspace depth function by the β-skeleton depth values. The goodness of approximation is measured by the sum of squares of error values. Secondly, computing the planar β-skeleton depth is reduced to a combination of some range counting problems. Using the existing results on range counting, the planar β-skeleton depth of a query point is approximated in $O(n\, poly(1/\varepsilon, \log n))$, $\beta \geq 1$. Convergence of β-skeleton depth functions when $\beta \to \infty$ is also proved theoretically and experimentally.