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Approximation of Data Depth Revisited

Data depth is a measure of centrality of $q \in \mathbb{R}^d$ with respect to a data set $S \subset \mathbb{R}^d$. Among various notions, two depth functions halfspace depth (Tukey, 1975) and β-skeleton depth (Yang, 2017) are considered in this study. The halfspace depth of $q \in \mathbb{R}^d$ with respect to $S \subset \mathbb{R}^d$ is the minimum portion of the elements of S which are located in one side of a halfspace passing through q. For $\beta \geq 1$, the β-skeleton depth of q with respect to S is the total number of β-skeleton influence regions that contain q, where each influence region is the intersection of two hyperballs obtained from a pair of points in S. Due to the hardness of computing the depth functions in some cases, approximation of depth functions is of interest. In this study, different methods are presented to approximate the halfspace and β-skeleton depth. First, an approximation technique is proposed to approximate the halfspace depth using the β-skeleton depth. Two dissimilarity measures based on the concepts of *fitting function* and *Hamming distance* are defined to train the halfspace depth function by the β-skeleton depth values. The goodness of approximation is measured by the sum of squares of error values. Secondly, computing the planar β-skeleton depth is reduced to a combination of some range counting problems. Using the existing results on range counting, the planar β-skeleton depth of a query point is approximated in $O(n\, poly(1/\varepsilon, \log n))$, $\beta \geq 1$. Convergence of β-skeleton depth functions when $\beta \to \infty$ is also proved theoretically and experimentally.