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An unbalanced Optimal Transport splitting scheme for general advection-reaction-diffusion problems

In this talk I will present a joint work with T.O. Gallouët and L. Monsaingeon. We show that unbalanced optimal transport provides a convenient framework to handle reaction and diffusion processes in a unified metric framework. We use a constructive method, alternating minimizing movements for the Wasserstein distance and for the Fisher-Rao distance, and prove existence of weak solutions for very degenerate reaction-diffusion-advection equations of the form $\partial_t \rho - div(\rho \nabla (V + p)) = \rho \Phi(p, x)$ where the potential drift V and the reaction term $\Phi(\cdot, x)$ are given. The diffusion nonlinearity can be of the Porous-Medium type $p = g_m(\rho) = \frac{m}{m-1}\rho^{m-1}$, and in the limit $m \to +\infty$ the diffusion degenerates in a Hele-Shaw type problem with hard constraint $\rho \leqslant 1$ and the maximal monotone graph relation: $p \in g_\infty(\rho) = 0$ if $0 \leqslant \rho < 1$ and $p \in g_\infty(\rho) = [0, +\infty$ if $\rho = 1$. Moreover, some numerical simulations will be presented.