M. WILSON, University of Vermont
Discretization with irregular grids

A set of functions $\{\psi_\gamma\}_{\gamma\in\Gamma} \subset L^2(\mathbf{R}^d)$ is called *almost-orthogonal* if there is a finite R_1 so that, for all finite subsets $\mathcal{F} \subset \Gamma$ and all linear combinations $\sum_{\gamma\in\mathcal{F}} \lambda_\gamma \psi_\gamma$,

$$\left\| \sum_{\gamma\in\mathcal{F}} \lambda_\gamma \psi_\gamma \right\|_{L^2(\mathbf{R}^d)} \leq R_1 \left(\sum_{\gamma\in\mathcal{F}} |\lambda_\gamma|^2 \right)^{1/2}.$$

If $\{\phi_\gamma\}_{\gamma\in\Gamma}$ is another almost-orthogonal family, with constant R_2, then

$$T(f) \equiv \sum_{\gamma\in\Gamma} \langle f, \psi_\gamma \rangle \, \phi_\gamma \qquad (1)$$

(where $\langle \cdot, \cdot \rangle$ is the inner product in $L^2(\mathbf{R}^d)$) converges unconditionally for all $f \in L^2(\mathbf{R}^d)$ to define a linear operator mapping $L^2(\mathbf{R}^d) \to L^2(\mathbf{R}^d)$, with bound $\leq R_1 R_2$. We look at familiar ("wavelet-like") almost-orthogonal families indexed over $\Gamma =$ the dyadic cubes. We show that they stay almost-orthogonal when they are replaced by their averages over rectangles defined by "irregular" grids (an operation we call irregular discretization), even though the discretized functions can have many jump discontinuities in bad places. We show that if the grids are fine enough, then the resulting discretizations of (1) provide good approximations to the operator T.