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Conservation laws of vorticity-type equations

Partial differential equations (PDE) of the form $\operatorname{div} \vec{N} = 0$, $\vec{N}_t + \operatorname{curl} \vec{M} = 0$ involving two vector functions $\vec{N}, \vec{M} \in \mathbb{R}^3$ that depend on t, x, y, z arise as subsets of PDE systems in various models, including the vorticity formulation of viscous and inviscid fluid dynamics, plasma physics (magnetohydrodynamics), and Maxwell's equations. We refer to these equations as "vorticity-type equations".

It is shown that vorticity-type equations have a special structure of a lower-degree (degree two) conservation law in $\mathbb{R}^4(t, x, y, z)$. Moreover, they form an abnormal PDE system, in the sense of possessing an identically vanishing differential consequence. Even though vorticity-type equations are not variational, a result similar to the Noether's second theorem holds: these equations admit an infinite-dimensional family of conservation laws involving an arbitrary function of all variables. Applications of these conservation laws and related results are discussed.