Analysis and applications of differential equations using symmetries, conservation laws, and integrability
Analyse et applications d'équations différentielles utilisant des symétries, les lois de la conservation et l'intégrabilité
(Org: **Stephen Anco** (Brock) and/et **Alexei Cheviakov** (Saskatchewan))

STEPHEN ANCO, Brock

GEORGE BLUMAN, University of British Columbia
Nonclassical analysis of the nonlinear Kompaneets equation

The nonlinear Kompaneets (NLK) equation describes the spectra of photons interacting with a rarefied electron gas. We exhibit five previously unknown classes of explicit time-dependent solutions (each class depending on initial conditions with two parameters) of the NLK equation. It is shown that these solutions cannot be found as invariant solutions using the classical Lie method (solutions obtained by Ibragimov (2010)) but are found using the nonclassical method. Interestingly, each of these new solutions can be expressed in terms of elementary functions. Three of these solution classes exhibit quiescent behaviour and the other two solution classes exhibit blow-up behaviour in finite time. As a consequence, it is shown that corresponding nontrivial stationary solutions are all unstable. This is joint work with Zhengzheng Yang and Shou-fu Tian. For details, see our paper with the same title in J. Eng. Math 84: 87-97 (2014)

ALEXEI CHEVIAKOV, University of Saskatchewan
Conservation law construction for the incompressible Mooney-Rivlin hyperelasticity model

An extended Kovalevskaya form is derived for the two-dimensional incompressible Mooney-Rivlin nonlinear hyperelasticity equations and is used to compute a complete set of local conservation laws of the model through the direct method. Conserved densities and fluxes of the conservation laws are derived, and their physical interpretation is discussed. Since the model admits a variational formulation, the equations are rewritten in the self-adjoint form. Computation of local conservation laws through the direct method applied to the self-adjoint form, as well as a conservation law computation through the local symmetry analysis and the Noether's first theorem, is performed. A correspondence between local variational symmetries and conservation law multipliers is illustrated. It is argued that even though it leads to more complicated forms of multipliers, the direct conservation law construction method applied to the Kovalevskaya form of the equations is a preferred systematic way of conservation law computations for complicated physical models of the type considered in this work, since it yields complete results, and naturally avoids singular multipliers. This is joint work with Simon St. Jean.

ALEXANDER ODESSKI, Brock University
Integrable structures in 3D hydrodynamic-type systems and differential geometry

We review the theory of 3D hydrodynamic-type systems and Whitham type hierarchies integrable by hydrodynamic reductions method. This approach to integrability is based on the so-called systems of Gibbons-Tsarev type. We explain that this integrable structure can be represented as a certain differential-geometric structure which is defined locally as a family of vector fields $g(p)=\sum_{i=1}^{m} g_i(p,v_1,...v_m)\frac{\partial}{\partial v_i}$ with commutation relations

$$[g(p_1),g(p_2)]=f(p_2,p_1)g'(p_1)-f(p_1,p_2)g'(p_2)+2f(p_2,p_1)_{p_1}g(p_1)-2f(p_1,p_2)_{p_2}g(p_2)$$

where

$$f(p_1,p_2)=\frac{1}{p_1-p_2}+O(1)$$

and

$g(p_2)(f(p_1,p_3))-g(p_1)(f(p_2,p_3)) = f(p_1,p_2)f(p_2,p_3)_{p_2}-f(p_2,p_1)f(p_1,p_3)_{p_1}+f(p_1,p_3)f(p_2,p_3)_{p_3}-f(p_2,p_3)f(p_1,p_3)_{p_3}+2f(p_2,p_3)f(p_1$

DMITRY PSHENITSIN, Brock

Conservation laws of incompressible magnetohydrodynamics

A classification of all local conservation laws of kinematic type is obtained for the system of magnetohydrodynamic equations governing incompressible viscous plasmas in which the dynamic and magnetic viscosities are constant. As one new result, conservation of cross-helicity is shown to extend from the ideal case to a special viscous case. A similar classification of conservation laws is derived under reductions by translation symmetries, axial rotation symmetries, and helical symmetries. The results yield many new conservation laws which are expected to be relevant in various physical applications of magnetohydrodynamics.

Authors: S. Anco. D. Pshentsin, T. Wolf

ELENA RECIO, Brock University

Multi-peakon solutions in a family of generalized Camassa-Holm equations

A new family of peakon equations depending on an arbitrary nonlinearity power p is introduced. This family generalizes the Camassa-Holm equation and shares one of its Hamiltonian structures. Multi-peakon solutions are derived for all powers p.

ANDY WAN, McGill University

Conservative discretization and long term stability

We review a recent method, called the multiplier method, on discretizing ODEs and PDEs so that their conservation laws are exactly preserved at the discrete level. In contrast to geometric numerical integrators, such as symplectic and variational integrators, the multiplier method is applicable for systems even without a symplectic or variational structure, such as dissipative problems. Moreover, we discuss the long term stability and preservation of topological properties of the multiplier method for ODEs.

Reference:

Andy T.S. Wan, Alexander Bihlo, and Jean-Christophe Nave, "The Multiplier Method to Construct Conservative Finite Difference Schemes for Ordinary and Partial Differential Equations", SIAM J. Numer. Anal., 54(1), 86–119, 2016

THOMAS WOLF, Brock

Computing Symmetries and Recursion Operators of Evolutionary Super-Sytstems

In the talk we discuss the step-by-step computation of nonlocal recursions for symmetry algebras of nonlinear coupled boson-fermion N = 1 supersymmetric systems by using the SsTools environment.