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Existence of Weak Solutions to a Non-Linear Dirichlet Problem

In analysis, it is common to consider the existence of weak solutions to equations which involve the Laplace operator. Often, the weak solutions to such equations live in Hilbert spaces, where results such as the Riesz-Representation Theorem can be easily applied to conclude existence of weak solutions (a well-known example is the Dirichlet problem for Poisson's equation). Proving the existence of weak solutions when working outside of a Hilbert space requires developing an entirely new set of techniques. In particular, my research has focused on weak solutions that live in specific Sobolev spaces that are not Hilbert spaces but, rather, are reflexive Banach Spaces. As such, this talk begins with a brief discussion of background material regarding functional analysis on reflexive Banach spaces. This will lead us to Minty's theorem, the tool used to conclude the existence of weak solutions to a non-linear Dirichlet problem. Specifically, Minty's theorem requires that an operator possess four properties: monotonicity, boundedness, hemicontinuity, and almost-coercivity. Each of these will be discussed in detail, making specific reference to the operator under scrutiny - an extension of the p-Laplacian.