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Maps with Memory

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Let $f : X \to X$ be a map. We want to consider a process, which is not a map, and represents situation when f on each step uses not only current information but also some information from the past. We define for current state x_n and $0 < \alpha < 1$:

$$x_{n+1} = f(\alpha x_n + (1-\alpha)x_{n-1})\,.$$

We are interested in something we could call an "invariant measure" of the process. We consider ergodic averages

$$A_f(x_0, x_{-1}) = \lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} f(x_i).$$

They are related to ergodic averages of the map $G : X \times X \to X \times X$ defined by

$$G(x,y) = (y, f(\alpha y + (1-\alpha)x)\,.$$

We considered the example where $f : [0,1] \to [0,1]$ is the tent map. Computer experiments suggest that G behaves in very different manners depending on α. We conjecture:

For $0 < \alpha < 1/2$ map G preserves absolutely continuous invariant measure.

For $\alpha = 1/2$ every point of upper half of the square ($y + x \geq 1$) has period 3 (except the fixed point $(2/3, 2/3)$). Every other point (except $(0,0)$) eventually enters the upper triangle.

For $1/2 < \alpha < 3/4$ point $2/3, 2/3$ is a global attractor for map G.

For $\alpha = 3/4$ every point of the interval $x+y = 4/3$ has period 2 (except the fixed point $(2/3, 2/3)$). Every other point (except $(0,0)$) is attracted to this interval.

For $3/4 < \alpha < 1$ map G preserves an SRB measure which is not absolutely continuous (supported on an uncountable union of straight intervals).