Ergodic Theory, Dynamical systems and Applications
Théorie ergodique, systèmes dynamiques et applications
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SANJEEVA BALASURIYA, University of Adelaide
Analogues of eigenvectors and their control in nonautonomous flows

In autonomous flows, eigenvectors of saddle fixed points are locally tangent to stable and unstable manifolds, which form important flow separators. Computing fixed points or instantaneous eigenvectors however does not identify local flow separators in nonautonomous flows. The correct analogues of eigenvectors are time-parametrised local tangents to time-varying stable and unstable manifolds at hyperbolic trajectory locations. These vectors indeed represent Oseledets spaces. For two-dimensional nearly-autonomous flows, these analogues are characterised in terms of a time-history of local velocity shear using definitions which can be applied to both finite- and infinite-time flows. The reverse question of whether one can force the Oseledets spaces to vary in any prescribed time-varying fashion is also addressed. A methodology for controlling the 'eigenvector' directions in this way by applying a local velocity shear is developed. The control method is verified for both smoothly and discontinuously time-varying directions using finite-time Lyapunov exponent fields, and excellent agreement is obtained.

ARNO BERGER, University of Alberta
Most finite-dimensional linear flows are Benford

This talk will present some recent results concerning the distribution of numerical data in dynamics, with an emphasis on data generated by finite-dimensional linear processes. Specifically, a necessary and sufficient condition will be discussed for every signal obtained from a linear process to either be trivial or else exhibit a strong form of Benford's Law (logarithmic distribution of significant digits).

ERIK BOLLT, Clarkson University
A measurable perspective on finite time coherence

The concept of coherent structures in a flow refers to notions of subsets of the flow which preserve some measurable quantity, despite the generally nonlinear flow: something simple embedded in the complexity. Our own perspective of shape coherent sets is defined in terms of flow that is locally as rigid body motions, and uncovered by investigating boundary curvature evolution. Key for unifying to other concepts of coherence is choice of measure interpreted across domains.

ELENA BRAVERMAN, University of Calgary
Stochastic difference equations with the Allee effect

Difference equations can describe population dynamics models, and, if there is no compensation for low population size, i.e. the stock recruitment is lower than mortality, the species goes to extinction, unless the initial size is large enough. This phenomenon is called the Allee effect. For a truncated stochastically perturbed equation $x_{n+1} = \max\{f(x_n) + l\chi_{n+1}, 0\}$ with $f(x) < x$ on $(0, m)$, which corresponds to the Allee effect, we observe that for very small perturbation amplitude, the eventual behavior is similar to a non-perturbed case: there is extinction for small initial values and persistence for large enough. As the amplitude grows, a sustainability interval of initial values arises and expands, such that with a certain probability, x_n sustains and possibly eventually stays in a low density area. Lower estimates for these probabilities are presented. If the upper bound of the sustainability interval is large enough, as the amplitude of perturbations grows, the Allee effect disappears: a solution persists for any positive initial value. This is a joint work with A.Rodkina (the University of West Indies, Kingston, Jamaica).

LAURENT CHARETTE, University of British Columbia
Reaction-Diffusion Mixed Modes on a Spherical Cap

Patterns in cotyledon formation in conifer embryos can be modeled using reaction-diffusion systems for underlying chemical morphogens. In order to approximate the geometry of a flattening embryo tip we use spherical cap domains. Nagata, Zangeneh and Holloway described the transition from the patternless state to a single mode, noted by a pair of integers (m, n), for a Brusselator system on such domains. We will use similar techniques to find the bifurcation diagram at the intersection of two single-mode transitions. In the simple case when both m values are non-zero we have a well documented double pitchfork bifurcation. If, however, one of the m is zero we have a more elaborate bifurcation diagram because we have to take into consideration the quadratic and cubic terms in the normal form. We present this codimension two bifurcation here and show solutions of a finite elements simulation that support our findings.

JACOPO DE SIMOI, University of Toronto
Decay of correlations in fast-slow partially hyperbolic systems

We show existence of an open class of partially hyperbolic smooth local diffeomorphisms of the two-torus which admit an unique SRB measure satisfying exponential decay of correlations and we obtain nearly optimal estimates for the rate of decay of correlations. In this talk, we will focus on the more dynamical aspects of the proof and discuss some related questions which still remain open. This is part of a joint project with Carlangelo Liverani.

GERDA DE VRIES, University of Alberta
A Model of Microtubule Organization in the Presence of Motor Proteins

Microtubules and motor proteins interact in vivo and in vitro to form higher-order structures such as bundles, asters, and vortices. In vivo, the organization of microtubules is connected directly to cellular processes such as cell division, motility, and polarization. To address questions surrounding the mechanism underlying microtubule organization, we have developed a system of integro-partial differential equations that describes the interactions between microtubules and motor proteins. Our model takes into account motor protein speed, processivity, density, and directionality, as well as microtubule treadmilling and re-organization due to interactions with motors. Our model is able to provide a quantitative and qualitative description of microtubule patterning. Simulations results show that plus-end directed motor proteins form vortex patterns at low motor density, while minus-end directed motor proteins form aster patterns at similar densities. Also, a mixture of motor proteins with opposite directionality can organize microtubules into anti-parallel bundles such as are observed in spindle formation.

MARLENE FRIGON, University of Montreal
Multiple solutions of problems with nonlinear first order differential operators

We present results establishing the existence of solutions to first order differential equations with nonlinear differential operators of the following form:

$$\phi(u(t)))' = f(t, u(t)) \quad \text{a.e. } t \in [0, T],$$

with periodic boundary value or the initial value conditions. Here ϕ is is an increasing nonlinear homeomorphism. Multiplicity results are also presented. Our results rely on the notion of upper and lower solutions and on the fixed point index theory.

PAWEL GORA, Concordia University, Montreal
Maps with Memory

Paweł Góra (Concordia University, Montreal) in collaboration with A. Boyarsky, P. Eslami, Zh. Li and H. Proppe.

Let $f : X \to X$ be a map. We want to consider a process, which is not a map, and represents situation when f on each step uses not only current information but also some information from the past. We define for current state x_n and $0 < \alpha < 1$:

$$x_{n+1} = f(\alpha x_n + (1 - \alpha)x_{n-1}) \,.$$

We are interested in something we could call an "invariant measure" of the process. We consider ergodic averages

$$A_f(x_0, x_{-1}) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} f(x_i).$$

They are related to ergodic averages of the map $G : X \times X \to X \times X$ defined by

$$G(x, y) = (y, f(\alpha y + (1-\alpha)x)\,.$$

We considered the example where $f : [0,1] \to [0,1]$ is the tent map. Computer experiments suggest that G behaves in very different manners depending on α. We conjecture:

For $0 < \alpha < 1/2$ map G preserves absolutely continuous invariant measure.

For $\alpha = 1/2$ every point of upper half of the square ($y + x \geq 1$) has period 3 (except the fixed point $(2/3, 2/3)$). Every other point (except $(0,0)$) eventually enters the upper triangle.

For $1/2 < \alpha < 3/4$ point $2/3, 2/3$ is a global attractor for map G.

For $\alpha = 3/4$ every point of the interval $x + y = 4/3$ has period 2 (except the fixed point $(2/3, 2/3)$). Every other point (except $(0,0)$) is attracted to this interval.

For $3/4 < \alpha < 1$ map G preserves an SRB measure which is not absolutely continuous (supported on an uncountable union of straight intervals).

TOBIAS HURTH, University of Toronto
Regularity for invariant densities of switching systems

Consider a finite collection D of smooth vector fields on $\mathbb{R}^n$. Given an initial vector field $u \in D$, we flow along u for a random time. Then, we switch to a new vector field that is randomly chosen from D. We flow along the new vector field for a random time and make another switch. Reiterating this procedure, we obtain a Markov process on $\mathbb{R}^n \times D$. If the associated semi-group admits an absolutely continuous invariant measure with density ρ, we can ask whether the projections $(\rho_u)_{u\in D}$ have smooth representatives. Another natural question is whether these projections have singularities. In dimension 1, under mild assumptions on D, singularities can only occur at critical points of the vector fields and the projections are smooth at noncritical points. In dimension 2, singularities may also form at noncritical points, which we will illustrate with a basic example. Whether and where singularities occur depends critically on the rate of switching. The talk is based on work with Yuri Bakhtin, Sean Lawley and Jonathan Mattingly.

TOMOKI INOUE, Ehime University
First return maps of random maps and invariant measures

We consider a family of transformations with a random parameter and consider a random dynamical system in which one transformation is randomly selected from the family and applied on each iteration. Such a process is called a random map. We consider the first return map of a random map and study how to construct an invariant measure of the original random map from an invariant measure of the first return map.

We also consider an application to one dimensional random maps with neutral fixed points.

SHAFIQUL ISLAM, University of Prince Edward Island (UPEI)
Stochastic perturbations and invariant measures of position dependent random maps via Fourier approximations

Let $T = \{\tau_1(x), \tau_2(x), \ldots, \tau_K(x); p_1(x), p_2(x), \ldots, p_K(x)\}$ be a position dependent random map which posses a unique absolutely continuous invariant measure $\hat{\mu}$ with probability density function $\hat{f}$. We consider a family $\{T_N\}_{N\geq 1}$ of stochastic perturbations T_N of the random map T. Each T_N is a Markov process with the transition density $\sum_{k=1}^{K} p_k(x) q^N(\tau_k(x), .)$, where

$q^N(x,.)$ is a doubly stochastic periodic and separable kernel. Using Fourier approximation, we construct a finite dimensional approximation P_N to a perturbed Perron-Frobenius operator. Let f_N^* be a fixed point of P_N. We show that $\{f_N^*\}$ converge in L^1 to $\hat{f}$.

FRANKLIN MENDIVIL, Acadia University
Iteration of time-dependent random functions

The dynamics of the iteration of some function is a classical area of study within dynamical systems, iteration of complex polynomials being a particularly striking example. It is no surprise then that adding randomness is a natural thing to do, particularly since many Markov chains can be viewed as a random iteration of functions.

In this talk we give a brief background on general results in the area of convergence theorems for random iteration of functions, with particular attention to the case of families of random contractions. After this we concentrate on some results on a simple model of time inhomogeneous random iteration. As is the case for Markov chains, allowing the dynamics to vary with time presents new complications.

ISRAEL NCUBE, Alabama Agricultural and Mechanical University
Stability of a certain quasi-polynomial

The talk deals with the direct analysis of a certain transcendental quasi-polynomial arising in modelling applications. Our main concern in the analysis is the distribution of characteristic roots in a suitable parameter space. As is well-known, characterisation of root distribution is essential in the description of stability of equilibria. Since the quasi-polynomial is transcendental, it is also true that the celebrated Hermite-Routh-Hurwitz criterion is not applicable. The classical Hermite-Biehler theorem gives necessary and sufficient conditions for the Hurwitz stability of a polynomial in terms of certain interlacing conditions. Inspired by the pioneering work of N.G. Cebatarev, L.S. Pontryagin generalised the Hermite-Biehler theorem to be able to handle some transcendental quasi-polynomials. Unfortunately, there are still many problems for which Pontryagin's method of root determination simply does not work. This talk will highlight some limitations of Pontryagin's method, before proceeding to describe an alternative approach.

ANATOLIY SWISHCHUK, University of Calgary
Delay Stochastic Dynamical Systems in Finance

The volatility process is an important concept in financial modeling. This process can be stochastic or deterministic. In quantitative finance, we consider the volatility process to be stochastic as it allows to fit the observed market prices under consideration, as well as to model the risk linked with the future evolution of the volatility, which deterministic model cannot. Heston model (1993), e.g., is one of the most popular stochastic volatility models in the industry as semi-closed formulas for vanilla option prices are available, few (five) parameters need to be calibrated, and it accounts for the mean-reverting feature of the volatility. In this talk we will focus on newly developed so-called delayed Heston model (2014) that significantly improve classical Heston model with respect to the market volatility surface fitting by 44%. In this model, we take into account not only current state of volatility at time t but also its past history over some interval $[t-\tau, t]$, where $\tau > 0$ is a constant and is called the delay. In this way, our model incorporates path-dependent history for volatility. We will show how to model and price variance and volatility swaps (forward contracts on variance and volatility for an underlying asset) for the delayed Heston model and how to hedge volatility swaps using variance swaps. Review of some other delay stochastic models in finance will be given as well.

JAMES A YORKE, Univ of Maryland
Computing Quasiperiodic Orbits

Quasiperiodic orbits occur in many dynamical systems, sometimes isolated and sometimes nested like the rings of an onion. By definition, a one-dimensional quasiperiodic orbit must have a change of variables that converts it into a pure rotation

$\theta \to \theta + rho \mod 1$ (measuring angles as fractions of a full rotation). We produce the change of variables and argue that they are real analytic (based on high precision arithmetic) in all the cases we examine. Examples include quasi-periodic orbits in the restricted circular three-body problem; the standard map; and a periodically forced Van der Pol equation and an example of a two-dimensional quasiperiodic orbit.

Collaborators, S. Das, Y. Saiki, E. Sander

HUAIPING ZHU, York University

Nilpotent singularities and oscillations in SIR type of compartmental models

The dynamics of SIR type of compartmental models can be complicated due the demographics and incidence mechanisms. In this talk, I will present SIR type of compartmental models with a standard incidence rate and a nonlinear recovery rate to reflect the impact of available resources of public health system especially the number of hospital beds. Cusp, focus and elliptic type of nilpotent singularities of of co-dimension 3 are discovered and analyzed in this three dimensional model. Complex dynamics of disease transmission including multi-steady states and multi-periodicity are revealed by bifurcation analysis. Large-amplitude oscillations found in our model provide a more reasonable explanation for disease recurrence. This is a joint work with Chunhua Shan.