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Flow through Layered Porous Media with Variable Permeability

In this work, we consider fluid flow through layered porous medium . Flow through layers is governed by Brinkman's equation. In layers 1 and 3, permeability is constant, in layer 2, permeability is taken as a function of y. Solution is obtained in layer 2 in terms of Airy's and the Nield-Koznetsov functions. Flow is governed by the following equations, subject to the indicated boundary conditions, which we derive and write in the following forms:

$$\mu_{ieff}\frac{d^2u_i^*}{dy^{*2}} - \frac{\mu_i}{K_i(y^*)}u_i^* + G = 0 \tag{1}$$

for $i =, 1, 2, 3$. In equation(1) $G = -\frac{dp}{dx}$ is the constant pressure gradient, $u_i^* = u_i^*(y^*)$, K_i, μ_i,μ_{ieff} are velocity, permeability, viscosity, effective viscosity of the fluid in the ith layer (respectively). The permeability K_1, K_3 are assumed to be constants of the form: $K_1 = aK_0$; for $0 < y^* < \eta H$. $K_3 = bK_0$;for $\xi H < y^* < H$. In layers 2: the permeability K_2 is assumed to be a function of y^* and given by $K_2(y^*) = \frac{ab(\eta-\xi)K_0H}{(b-a)y^*+(a\eta-b\xi)H}$; where K_0 is a reference constant permeability, a and b are constants to be selected, η and ξ are parameters that determine the thickness of each layer. The above equations are to be rewritten in dimensionless form and solved subject to the conditions of no-slip at the solid walls (y = 0 and y = 1), velocity and shear-stress continuity at the interfaces between layers,$y = \eta$ and $y = \xi$. We will show that the solution will have the following forms

$$u_1(y) = c_1exp(\lambda_1 y) + d_1exp(-\lambda_1 y) + \frac{1}{M_1\lambda_1^2} \tag{2}$$

$$u_2(y) = c_2Ai(\lambda_2[(b-a)y + (a\eta - b\xi)]) + d_2Bi(\lambda_2[(b-a)y + (a\eta - b\xi)]) + \frac{\pi}{M_2(b-a)^2\lambda_2^2}Ni(\lambda_2[(b-a)y + a\eta - b\xi)]). \tag{3}$$

$$u3(y) = c_3exp(\lambda_3 y) + d_3exp(-\lambda_3 y) + \frac{1}{M_3\lambda_3^2}. \tag{4}$$

where $\lambda_1 = \frac{1}{\sqrt{aDaM_1}}$,$\lambda_2 = \frac{1}{\sqrt[3]{ab(b-a)^2DaM_2(\eta-\xi)}}$,$\lambda_2 = \frac{1}{\sqrt{bDaM_3}}$, $Da = \frac{K_0}{H^2}$, Ai, Bi are Airy's functions, and Ni is the Nield-Koznetsov function.