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On the approximation with Jacobi weights

We will discuss some open problems and results in the the approximation with Jacobi weights

$$q_{\alpha,\beta}(x) := (1-x)^{\alpha}(1+x)^{\beta}$$

and their generalizations. Say, for the error

$$E_n^*(f) := \inf_{P_n} \|(f-P_n)q_{-1/2,-1/2}\|_{C[-1,1]}$$

of the best uniform weighted approximation by algebraic polynomials P_n of degree $< n$ of a continuous on $[-1,1]$ function $f \in$ Lip1 (that is $|f(x') - f(x'')| \le |x' - x''|, \forall x', x'' \in [-1,1]$) the estimate

$$nE_n^*(f) \le c$$

is well known. However the exact value of the absolute constant c is not known.

Recently the asymptotically exact estimates for the generalized Lebesgue constants of the Fourier-Jacobi sums in the weighted integral norm are obtained.

For a domain $D \subset \mathbf{C}$ with piecewise smooth boundary an analog of the Jacobi weight is

$$q_{\alpha_1,\dots,\alpha_s}(z) := |z_1 - z|^{\alpha_1} \dots |z_s - z|^{\alpha_s},$$

where s is the number of the angular points z_j. For such domains and weights complete analogs of the estimates, taking place for the interval, are proved.

Finally, we will discuss the shape preserving approximation with Jacobi weights.

This is joint work with O. V. Motorna.