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**Plenary Speakers**  
**Conférences plénières**

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**BELA BOLLOBAS**, Cambridge and Memphis

*Percolation, Bootstrap Percolation, and Random Monotone Cellular Automata*

*Percolation theory*, founded by Broadbent and Hammersley in 1957, and *r-neighbour bootstrap percolation*, introduced by Chalupa, Leath and Reich in 1979, have much in common. Thus, on the infinite lattice  $\mathbb{Z}^d$  we may start the same way, by selecting vertices of  $\mathbb{Z}^d$  with the same probability  $p$  to obtain a random set  $A_0$ . In percolation a basic question is whether  $A_0$  *percolates*, i.e. the subgraph induced by it has an infinite component; the infimum of those probabilities is the *critical probability*  $p_c$ . In *r-neighbour bootstrap percolation* the set  $A_0$  is only the beginning: considering the vertices in it *infected*, the infection spreads in a very simple way: infected vertices remain infected for ever, and if at any stage a vertex has at least  $r$  infected neighbours then itself becomes infected. This system *percolates* if each site is infected eventually. The critical probability  $p_c$  is defined as in percolation.

In percolation much work has been done on the critical probability, since it is strictly between 0 and 1 in every non-trivial model; however, in *r-neighbour bootstrap percolation* the critical probability is of no interest, since every lattice has critical probability 0 or 1.

Recently, Smith, Uzzell and the lecturer introduced a wide-ranging extension of *r-neighbour bootstrap percolation*: they initiated the study of completely general monotone, local, and homogeneous cellular automata in a random environment. Among other results, they classified these *U-percolation models* into three types, and proved results about the phase transition in two of them. The phase transition in the third type has been clarified by Balister, Przykucki, Smith and the lecturer: in particular, they have shown that in  $\mathbb{Z}^2$  these processes have non-trivial critical probabilities. These results have reopened the study of critical probabilities in ‘generalized bootstrap processes’ on  $\mathbb{Z}^2$ .

In the lecture I shall survey some classical results in percolation and bootstrap percolation, and will sketch some of the recent results on monotone cellular automata obtained by Balister, Duminil-Copin, Gunderson, Holmgren, Morris, Przykucki, Smith, Uzzell, and myself.

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**MARK LEWIS**, University of Alberta

*The mathematics behind biological invasion processes*

Models for invasions track the front of an expanding wave of population density. They take the form of parabolic partial differential equations and related integral formulations. These models can be used to address questions ranging from the rate of spread of introduced invaders and diseases to the ability of vegetation to shift in response to climate change.

In this talk I will focus on scientific questions that have led to new mathematics and on mathematics that have led to new biological insights. I will investigate the mathematical and empirical basis for multispecies invasions, for accelerating invasion waves, and for nonlinear stochastic interactions that can determine spread rates.

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**JAMES MAYNARD**, CRM, Université de Montréal

*Small Gaps between primes*

It is believed that there should be infinitely many pairs of primes which differ by 2; this is the famous twin prime conjecture. More generally, it is believed that for every positive integer  $m$  there should be infinitely many sets of  $m$  primes, with each set contained in an interval of size roughly  $m \log m$ . Although proving these conjectures seems to be beyond our current techniques, recent progress has enabled us to obtain some partial results. We will introduce a refinement of the ‘GPY sieve method’ for studying these problems. This refinement will allow us to show (amongst other things) that  $\liminf_n (p_{n+m} - p_n) < \infty$  for any integer  $m$ , and so there are infinitely many bounded length intervals containing  $m$  primes.

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**THOMAS RANSFORD**, Université Laval

*Computation of analytic capacity and the subadditivity problem*

Analytic capacity measures the size of compact plane sets from the point of view of certain aspects of complex analysis. It was first introduced in the 1940's in connection with the so-called Painlevé problem of characterizing removable singularities of bounded holomorphic functions. Later, in the 1960's, it played a key role in the solution of fundamental problems in rational approximation. The last twenty years have seen significant advances in the understanding of analytic capacity, one of the most striking breakthroughs being the positive solution to the long-standing conjecture that analytic capacity is semi-additive (the capacity of the union of two sets is bounded by  $C$  times the sum of their individual capacities, where  $C$  is a universal constant). However, whether analytic capacity is subadditive (can we take  $C=1$ ?) still remains an open problem.

The plan for the talk is (1) a brief history of analytic capacity and its applications, (2) a practical method for rigorous computation of analytic capacity, and (3) the hunt for a counterexample to the subadditivity problem. Parts (2) and (3) are based on joint work with Malik Younsi.