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On Littlewood Polynomials with Prescribed Number of Zeros Inside the Unit Disk

We investigate the numbers of complex zeros of Littlewood polynomials $p(z)$ (polynomials with integer coefficients $\{-1, 1\}$) inside or on the unit circle $|z| = 1$, denoted by $N(p)$ and $U(p)$, respectively. Two types of Littlewood polynomials are considered: Littlewood polynomials with one sign change in the sequence of coefficients and Littlewood polynomials with one negative coefficient.

We obtain explicit formulas for $N(p)$, $U(p)$ for polynomials $p(z)$ of these types. In particular, we show that if $n + 1$ is a prime number, then for each integer k , $0 \leq k \leq n - 1$, there exists a Littlewood polynomial $p(z)$ of degree n with $N(p) = k$ and $U(p) = 0$. Furthermore, we describe some cases when the ratios $N(p)/n$ and $U(p)/n$ have limits as $n \rightarrow \infty$ and find the corresponding limit values.