Symplectic geometry and equivariant topology
Géométrie symplectique et topologie équivariante
(Org: **Thomas Baird** (Memorial) and/et **Derek Krepski** (Manitoba))

MAXIME BERGERON, University of British Columbia
Kempf-Ness Theory and Character Varieties

Let V be a complex vector space equipped with an action of a reductive algebraic group $G \subset \mathrm{GL}(V)$. If $K \subset G$ is a maximal compact Lie subgroup then there is always a natural symplectic structure on V for which the action of K is Hamiltonian. In this setting we can consider two "quotients" of V: the geometric invariant theory quotient of V by G and the symplectic reduction of V by K. Kempf-Ness theory describes striking connections between these two worlds and in this talk I will explain how these ideas can be adapted to understand the topology of $G-$character varieties of finitely generated nilpotent groups.

PETER CROOKS, University of Toronto
The Torus-Equivariant Cohomology of Nilpotent Orbits in Semisimple Lie Algebras

Let G be a connected, simply-connected complex semisimple group with Lie algebra $\mathfrak{g}$. An adjoint G-orbit is called nilpotent if it lies in the nilpotent cone of $\mathfrak{g}$. This talk aims to introduce nilpotent orbits in the context of equivariant topology and geometry.
We will begin by introducing nilpotent orbits as objects studied at the interface of symplectic geometry, representation theory, and algebraic geometry. We will subsequently restrict our attention to two distinguished nilpotent G-orbits, the regular and minimal orbits. I will present some recent work on the equivariant cohomology of each orbit for the action of a maximal torus of G.

JONATHAN FISHER, University of Toronto
Quivers and Higgs bundles

Quiver representations may be used to construct a large class of holomorphic Poisson varieties. We show that varieties associated to star-shaped quivers may be mapped into Hitchin systems over $\mathbf{P}^1$, giving them the structure of algebraic completely integrable systems. In the case of rank 2 bundles, this gives an interesting relationship between Hitchin systems and the moduli space of polygons in $\mathbf{R}^3$. This is based on joint work with Steven Rayan.

SEAN FITZPATRICK, Western
Higher rank Boothby-Wang fibrations

The Boothby-Wang theorem gives the conditions under which a contact manifold is a prequantum circle bundle over a symplectic manifold. Since there are many different ways to characterize contact geometry, it is perhaps not surprising that there are several inequivalent generalizations to distributions of higher corank. The definition of a contact metric structure provides one such generalization, known as an almost S-structure, to which a generalized Boothby-Wang theorem applies: I will show that a manifold equipped with a regular almost S-structure is a principal torus bundle over a symplectic manifold. Time permitting, I will describe other properties of such structures, including symplectization, associated Jacobi structures, and connections to CR geometry.

MATTHIAS FRANZ, University of Western Ontario
Big polygon spaces produce maximal syzygies in equivariant cohomology

Let $T = (S^1)^r$ be a torus. We present a new class of compact orientable T-manifolds, called "big polygon spaces". Like polygon spaces, which appear as their fixed point sets, they depend on a length vector $\ell \in \mathbb{R}^r_{\geq 0}$. Although the equivariant cohomology of a big polygon space $X(\ell)$ is never free over $H^*(BT)$, one can observe interesting phenomena for suitable ℓ. In particular, $H_T^*(X(\ell))$ can be descibed by the "GKM method", and the equivariant Poincaré pairing for X can be perfect. The existence of such T-manifolds was unknown so far. More generally, $H_T^*(X(\ell))$ can be a syzygy of any order less than $r/2$ over $H^*(BT)$, which shows that a bound on the syzygy order obtained by Allday–Franz–Puppe is sharp.

MARCO GUALTIERI, University of Toronto
Log affine manifolds and symplectic geometry

I will describe a construction of log affine manifolds which is useful for studying symplectic and generalized complex structures.

ERIC HARPER, McMaster University
SU(N) Casson-Lin invariants for links in S^3

In 1992, X.-S. Lin introduced a Casson-type invariant $h(K)$ of knots $K \subset S^3$ via a signed count of conjugacy classes of irreducible $SU(2)$ representations of the knot group $\pi_1(S^3 - K)$ where all meridians of K are represented by trace-free $SU(2)$ matrices. Lin showed that $h(K)$ equals one-half the knot signature of K. With N. Saveliev, we defined an invariant of 2-component links $L \subset S^3$ using a construction analogous to Lin's. The invariant $h(L)$ is a signed count of conjugacy classes of certain projective $SU(2)$ representations of the link group $\pi_1(S^3 - L)$. We showed that $h(L)$ equals the linking number. In a recent joint work with H. U. Boden, we introduce invariants for n-component links L in S^3 where $n \geq 2$. The invariants are denoted $h_{N,a}(L)$ where $a = (a_1, \ldots, a_n)$ is an n-tuple of integers and each a_i labels the i-th component of the link. They are defined as a signed count of conjugacy classes of certain projective $SU(N)$ representations of $\pi_1(S^3 - L)$. In this talk, we will outline their construction, give a vanishing result for split links, and discuss some preliminary computations.

SHENGDA HU, Wilfrid Laurier University
Generalized holomorphic bundles, Part II

This is the second part of a joint talk with Ruxandra Moraru.

We discuss an analogue of the Hermitian-Einstein equations for generalized Kähler manifolds. We also introduce a notion of stability for generalized holomorphic bundles on generalized Kähler manifolds, and establish a Kobayashi-Hitchin-type correspondence between stable bundles and solutions of the generalized Hermitian-Einstein equations.

LISA JEFFREY, University of Toronto
Intersection cohomology of universal imploded cross-section

(Joint work with Nan-Kuo Ho)

If G is a compact Lie group, and G acts semifreely on a Hamiltonian G-space, then the preimage of the Lie algebra of the maximal torus contains only finitely many points in each orbit. More generally to get a space with this property we define the imploded cross-section of a Hamiltonian G-space by quotienting each orbit by the commutator subgroup of the stabilizer. The universal imploded cross-section is the imploded cross-section of the cotangent bundle of G – it can be used to construct the imploded cross-section of a general Hamiltonian G-manifold.

For SU(2) the universal imploded cross-section is a complex vector space of dimension 2, so its topology is trivial. In general the universal imploded cross-section is singular, but topological invariants distinguishing it from a point are not known. We compute the intersection cohomology of the universal imploded cross-section of SU(3), and show that it is nontrivial.

RUXANDRA MORARU, University of Waterloo
Generalized holomorphic bundles, Part I

This is the first part of a joint talk with Shengda Hu.

Generalized holomorphic bundles, introduced by Gualtieri in 2004, are the analogues of holomorphic vector bundles in the generalized geometry setting. For some generalized complex structures, these bundles correspond to co-Higgs bundles, flat bundles or Poisson modules. I will give an overview of what is known about generalized holomorphic bundles, and describe their moduli spaces in some specific examples.

BRENT PYM, University of Oxford
Categorified isomonodromic deformations via Lie groupoids

Given a meromorphic connection on a Riemann surface, one can seek deformations of the connection in which the locations of the poles are varied but the monodromy and Stokes data are held fixed. The solutions of this "isomonodromy problem" are unique up to isomorphism and can often be written explicitly in terms of special functions, such as the Painlevé transcendents. I will describe joint work with Marco Gualtieri in which we categorify this picture, promoting the classical special functions to functors using the theory of Morita equivalence for Lie groupoids. The Morita equivalences in question are themselves the solutions of an isomonodromy problem—the one for which the initial condition is the meromorphic projective connection provided by the uniformization theorem.

DAN RAMRAS, Indiana University-Purdue University Indianapolis
Moduli spaces of representations

I'll discuss recent work regarding the homotopy groups of moduli spaces of representations. For fundamental groups of Riemann surfaces, this work leads to a computation of the fundamental group of the GL(n) moduli space, as well as a complete understanding of the homotopy type of the stable moduli space of SU(n) representations. Connections to Goldman's symplectic form and the associated Ramadas-Singer-Weitsman line bundle will also be discussed.

SOUMEN SARKAR, University of Regina
Complex cobordism of quasitoric orbifolds

We construct manifolds and orbifolds with quasitoric boundary. We show that these manifolds and orbifolds with boundary has a stable complex structure. These induce explicit (orbifold) complex cobordism relations among quasitoric man- ifolds and orbifolds. In particular, we show that a quasitoric orbifold is complex cobordant to some copies of fake weighted projective spaces. The main tool is the theory of toric topology.

YANLI SONG, University of Toronto
The Cubic Dirac Operator and Geometric Quantization

In this talk, I will reformulate the quantization of Hamiltonian G-spaces as push-forward of the Dirac element in K-homology of crossed product of C^*-algebras. After localization, we can artificially construct a Dirac operator which is a mixture of algebraic cubic Dirac operator and geometric Spin^c-Dirac operator. This will reduce the quantization commutes with reduction theorem to a easy case. By small calculation, we obtain a simplified proof to the theorem. I will also explain how to apply this method to the quasi-Hamiltonian G-spaces.

JORDAN WATTS, University of Illinois at Urbana-Champaign
Coarse Moduli Spaces of Stacks over Manifolds

Consider a Lie group acting properly on a manifold. In the literature, the orbit space of the action has been equipped with various definitions of "smooth structure" for the purpose of extending differential geometry/topology to this space. Examples include differential structures and diffeologies. However, these structures often forget certain invariants induced by the group action. Stacks, on the other hand, encode many of these invariants into the so-called quotient stack.

In this talk, I will show how any stack over manifolds has an underlying coarse moduli space equipped with a diffeology which, in the case of a geometric stack, corresponds to the orbit space of a representative Lie groupoid equipped with the quotient diffeology. Moreover, there is a fully faithful functor from diffeological spaces into stacks. This gives us a unifying category in which we can directly compare, in the case of a Lie group action for instance, information encoded by the diffeology versus information encoded by the quotient stack. Time permitting, I will give an example of one such invariant.

This is joint work with Seth Wolbert.