Gröbner Bases and Computer Algebra
Bases de Gröbner et Algèbre Informatique

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Gröbner-Shirshov bases for semirings

We establish Gröbner-Shirshov bases theory for semirings and commutative semirings. As applications, we obtain Gröbner-Shirshov bases and A. Blass's (1995) and M. Fiore–T. Leinster's (2004) normal forms of the semirings $\mathbb{N}[x]/(x = 1 + x + x^2)$ and $\mathbb{N}[x]/(x = 1 + x^2)$, correspondingly. This is a joint work with L.A. Bokut and Qiuhui Mo.

MICHAEL KINYON, University of Denver
Loop Theory and Automated Deduction

In the past 15 years, automated deduction tools such as Prover9 and finite model builders such as Mace4 have increasingly played an important role in finding new results in loop theory. A *loop* is a quasigroup with an identity element, that is, it is a set Q with a binary operation $\cdot$ such that for each $a, b \in Q$, the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions $x, y \in Q$. (Besides groups, probably the class of loops best known to people outside the field are Moufang loops.)

In this talk, I will focus on a major automated deduction project in loop theory which is pushing all the available software to its limits: the AIM project (AIM = Abelian Inner Mappings). The problem is to find the correct loop theoretic generalization of the following classical fact from group theory: a group is nilpotent of class at most 2 if and only if its inner automorphism group is abelian. Progress toward the main conjectures in the loop setting has been slow but substantial, and I will concentrate on recent work.

ANATOLY KORYUKIN, Sobolev Institute of mathematics, Novosibirsk, Russia
Gröbner-Shirshov bases of the Lie algebras A_n^+, B_n^+, C_n^+, D_n^+

Over a field of characteristic 0, the linear reduced bases and reduced Gröbner-Shirshov bases (RGSB) are calculated of the Lie algebras A_n^+, B_n^+, C_n^+, D_n^+ for an arbitrary ordering of the generators corresponding to the simple roots.

Previously, linear reduced bases and RGSBs were calculated only for a specific ordering of the generators. Within the statement of the problem under consideration, the generators of a Lie algebra are fixed, while they are ordered in an arbitrary way, and we analyze an arbitrary basis of the $n!$ bases and the RGSB determined by each of them (here n is the number of generators).

An approach is proposed which combines the systems of roots, the linear reduced bases of the Lie algebras A_n^+, B_n^+, C_n^+, D_n^+, and their RGSBs.

XUAN LIU, Western University
Algorithms for finding Lie superalgebra structure of regular super differential equations

Super differential equations are non-commutative generalization of conventional differential equations, and arise naturally in some physics field theories. Exactly solving conventional differential equations is often difficult and similarly solving super differential equations maybe difficult or impossible. Lie supersymmetry is a generalization of Lie symmetry to super differential equations and the infinitesimal form of the supersymmetries satisfy a supersymmetry defining system.

I show how to find the structure (commutator table) of Lie super algebra of symmetries of sufficiently regular super differential equations without solving their supersymmetry defining system. I will use two examples to show the new method which uses existing commutative Maple commands for such non-commutative calculations.

SARA MADARIAGA, Department of Mathematics and Statistics, University of Saskatchewan
Jordan quadruple systems

In this joint work with Prof. Murray R. Bremner we define Jordan quadruple systems by the polynomial identities of degrees 4 and 7 satisfied by the Jordan tetrad a,b,c,d = abcd + dcba as a quadrilinear operation on associative algebras and find special identities in degree 10. We also introduce four infinite families of finite dimensional Jordan quadruple systems, and construct the universal associative envelope for the smallest system in each family.

MARC MORENO MAZA, The university of Western Ontario
On Fulton's Algorithm for Computing Intersection Multiplicities in Higher Dimension

Remarkably, and as pointed out by Fulton in his *Intersection Theory*, the intersection multiplicities of the plane curves $V(f)$ and $V(g)$ satisfy a series of 7 properties which *uniquely* define $I(p; f, g)$ at each point $p \in V(f, g)$. Moreover, the proof of this remarkable fact is constructive, which leads to an algorithm, that we call Fulton's Algorithm.

This construction, however, does not generalize to n polynomials $f_1, \ldots, f_n$ (generating a zero-dimensional of $k[x_1, \ldots, x_n]$, for an arbitrary field k) for $n > 2$. Another practical limitation, when targeting a computer implementation, is the fact that the coordinates of the point p must be in the base field k. Approaches based on standard or Groebner bases suffer from the same limitation.

In this work, we adapt Fulton's Algorithm such that it can work at any point of $V(f, g)$, rational or not. In addition, and under genericity assumptions, we add an 8-th property to the 7 properties of Fulton, which ensures that these 8 properties uniquely and constructively define $I(p; f_1, \ldots, f_n)$ at any $p \in V(f_1, \ldots, f_n)$. The implementation of this 8-th property has lead us to a new approach for computing tangent cones that do not involve standard or Groebner bases. In fact, all our algorithms simply rely on the theory of regular chains and are implemented in the RegularChains library in Maple.

This is a joint work with Steffen Marcus, Eric Schost and Paul Vrbik.

HOSSEIN IRAGHI MOGHADDAM, University of Manitoba
On Semigroups admitting Commutators or Conjugates

In this joint work with R. Padmanabhan, we define conjugates and commutators on a cancellative semigroup S, and then we introduce some commutator laws and prove certain equivalent statements when S admits comutators. Also when S only admits conjugates we prove that S can be embedded into a group.

MICHAEL MONAGAN, SFU

R. PADMANABHAN, University of Manitoba
Algebra and Geometry with Computers

Sir Roger Penrose once remarked that computers are intrinsically limited, compared to humans, when it comes to the doing of mathematics. Even those who think that such things can be proved may be interested in the empirical question: what kind of mathematics can computers do? Can a computer reason logically like humans and prove new theorems? A partial answer can be given using modern automated reasoning software such as Otter and Prover9. Otter, developed by William McCune at the Argonne National Laboratory, is the first widely used high-performance theorem prover. It is based on first-order inference rules, substitution principles, unification etc. Otter has since been replaced by Prover9, which is paired with Mace4, a counter-example generator. Both are available free for Mac OSX, Windows and the Unix platforms. In this presentation, we plan to give a brief survey of this relatively new area of experimental mathematics and give a list of "new" theorems proved with the help of these theorem-provers. In particular, we plan to show some live demonstrations of automated deduction in the following areas of algebra and geometry: 1. Ring theory - commutativity theorems (in collaboration with Yang Zhang) 2. Cancellation Semigroup Conjecture (in collaboration with Iraghi Moghaddam) 3. Inverse semigroups (in collaboration with Michael Kinyon and J. Araujo) 4. Lattice theory - uniquely complemented lattices and Huntington laws 5. Projective planes as groups (using Maple routines) 6. Cayley-Bacharach implications (in collaboration with Bob Veroff).

J.D. PHILLIPS, Northern Michigan University
On Bruck Loops

We present some recent results on (left) Bruck loops, emphasizing the enigmatic behavior of the right nucleus.

GREG REID, Western University
Approximate symmetry analysis

There are effective symbolic algorithms for determining the Lie symmetry algebra structure of transformations leaving invariant algebraic systems of partial differential equations. Such algorithms depend on generalization of Grobner basis methods to differential systems. Direct application of such methods to models involving approximate data is unstable and usually yields uninteresting results, since the even the tiniest error, means that such methods can at most detect generic symmetries of nearby models, rather than exceptional symmetry rich models.

In this talk we discuss progress in the numerical detection of symmetry rich models which is stable to small errors in data. This builds on earlier results where we showed that the dimension of symmetry algebras of nearby symmetry rich models could be detected. Our method is to couple the geometrical technique of involutive systems with the singular value decomposition to the defining systems for such approximate symmetries. This is joint work with Tracy Huang and Ian Lisle.

WILLIAM SIT, The City College of The City University of New York
On Rota's Problem, Differential Type and Rota-Baxter Type Algebras

In this talk, I will recall Rota's Problem for linear operators on associative algebras, and discuss how computer algebra methods lead to a computed list of possible operators and the related algebras: one class being "differential type" and another being "Rota-Baxter type". If time allows, I will report on some recent developments on the list and propose some related research projects.

This is joint work with Li Guo (Rutgers University at Newark, USA) and Ronghua Zhang (Yunnan University, Kunming, China).

XIANGUI ZHAO, University of Manitoba
Gelfand-Kirillov dimension of modules over differential difference algebras

Gelfand-Kirillov dimension is a very useful and powerful tool for investigating noncommutative algebras and their modules. Differential difference algebras, introduced by Mansfield and Szanto in 2003, arose from the calculation of symmetries of discrete systems. In this talk, we will demonstrate how to use Gröbner-Shirshov basis theory to compute the Gelfand-Kirillov dimension of a finitely generated module over a differential difference algebra.