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On Fulton's Algorithm for Computing Intersection Multiplicities in Higher Dimension

Remarkably, and as pointed out by Fulton in his *Intersection Theory*, the intersection multiplicities of the plane curves $V(f)$ and $V(g)$ satisfy a series of 7 properties which *uniquely* define $I(p; f, g)$ at each point $p \in V(f, g)$. Moreover, the proof of this remarkable fact is constructive, which leads to an algorithm, that we call Fulton's Algorithm.

This construction, however, does not generalize to n polynomials $f_1, \ldots, f_n$ (generating a zero-dimensional of $k[x_1, \ldots, x_n]$, for an arbitrary field k) for $n > 2$. Another practical limitation, when targeting a computer implementation, is the fact that the coordinates of the point p must be in the base field k. Approaches based on standard or Groebner bases suffer from the same limitation.

In this work, we adapt Fulton's Algorithm such that it can work at any point of $V(f, g)$, rational or not. In addition, and under genericity assumptions, we add an 8-th property to the 7 properties of Fulton, which ensures that these 8 properties uniquely and constructively define $I(p; f_1, \ldots, f_n)$ at any $p \in V(f_1, \ldots, f_n)$. The implementation of this 8-th property has lead us to a new approach for computing tangent cones that do not involve standard or Groebner bases. In fact, all our algorithms simply rely on the theory of regular chains and are implemented in the RegularChains library in Maple.

This is a joint work with Steffen Marcus, Eric Schost and Paul Vrbik.