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*Representation of integers base  $d$  with digits  $0, 1, \dots, q - 1$*

Let  $d$  and  $q$  be positive integers, and consider representing a positive integer  $n$  with base  $d$  and digits  $0, 1, \dots, q - 1$ . Clearly if  $q < d$ , then not all positive integers can be represented. If  $q = d$ , every positive integer can be represented in exactly one way. If  $q > d$ , then there may be multiple ways of representing the integer  $n$ . For example, if  $d = 2$  and  $q = 3$  we might represent 6 as  $110 = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$  as well as  $102 = 1 \cdot 2^2 + 0 \cdot 2^1 + 2 \cdot 2^0$ . (This list is not complete.) Let  $f_{d,q}(n)$  be the number of representations of  $n$  with base  $d$  and digits  $0, 1, \dots, q - 1$ . In this talk we will look at the asymptotics of  $f_{d,q}(n)$  as  $n \rightarrow \infty$ . This depends in a rather strange way on the Generalized Thue-Morse sequence. Many of the results are computationally/experimentally true, although no proofs are known.