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On the total perimeter of convex bodies in a container

For two convex bodies, C and D, consider a packing S of n positive homothets of C contained in D. We estimate the total perimeter of the bodies in S, denoted per(S), in terms of n. When all homothets of C touch the boundary of the container D, we show that either per$(S) = O(\log n)$ or per$(S) = O(1)$, depending on how C and D "fit together," and these bounds are the best possible apart from the constant factors. Specifically, we establish an optimal bound per$(S) = O(\log n)$ unless D is a convex polygon and every side of D is parallel to a corresponding segment on the boundary of C (for short, D is *parallel to* C). When D is parallel to C but the homothets of C may lie anywhere in D, we show that per$(S) = O((1+\text{esc}(S)) \log n/ \log\log n)$, where esc$(S)$ denotes the total distance of the bodies in S from the boundary of D. Apart from the constant factor, this bound is also the best possible. (Joint work with Adrian Dumitrescu)