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A Doubling game avoiding three term arithmetic progressions

We consider the following Doubling game on a heap of tokens: the next player removes any positive number of tokens, *or* first doubles the previous player's removal and then removes any number of tokens from the remaining heap. A player who is unable to double the previous player's removal wins. Hence, you never want to remove more than half of the remaining heap. The starting position is special, because the previous removal is empty. We prove that the starting position is in P if and only if the heap size does not contain any 2s in its ternary expansion. This sequence was first studied by Szekeres, Erdős and Turán, obtained by greedily avoiding non-trivial arithmetic progressions of the form $2y = x + z$, on the nonnegative integers. It cannot be simulated by standard (invariant) subtraction games on one heap of tokens. Thus we resolve a new class of sequences as winning positions of impartial games. Our Doubling game has equivalent formulations on hyper graphs, as board games and as comply games with vast generalizations on finite numbers of heaps. It can also be played as a Multiplication game, with potential in educational games/recreational mathematics.