Commutative Algebra and Combinatorics
Combinatoire et algèbre commutative

(Org: **Jennifer Bierman** (Lakehead), **Sara Faridi** (Dalhousie), **Andrew Hoefel** (Queens) and/et **Adam Van Tuyl** (Lakehead))

ALI ALILOOEE, Dalhousie university

When is a Square-Free Monomial Ideal of Linear Type ?

In 1995 Villarreal gave a combinatorial description of the presentation ideals of Rees algebras of quadratic square free monomial ideals. His description was based on the concept of closed even walks in a graph. In this paper we will generalize his results for hypergraphs. Our approach is based on giving a definition of closed even walks in a simplicial complex. We apply this combinatorial method to square free monomial ideals of higher dimension.

MASSIMO CABOARA, University of Pisa

Reducing the size and number of linear programs in a dynamic Groebner basis algorithm.

Joint work with J.Perry.

The dynamic algorithm to compute a Gröbner basis is nearly twenty years old, yet it seems to have arrived stillborn; aside from two initial publications, there have been no published followups. One reason for this may be that, at first glance, the added overhead seems to outweigh the benefit; the algorithm must solve many linear programs with many linear constraints. This talk describes two methods that reduce both the size and number of these linear programs.

GIULIO CAVIGLIA, Purdue University

Betti tables of p-Borel-fixed ideals

We provide a counter-example to a conjecture of K. Pardue [Thesis, Brandeis University, 1994.], which asserts that if a monomial ideal is p-Borel-fixed, then its $\mathbb{N}$-graded Betti table, after passing to any field does not depend on the field. More precisely, we show that, for any monomial ideal I in a polynomial ring S over the ring $\mathbb{Z}$ of integers and for any prime number p, there is a p-Borel-fixed monomial S-ideal J such that a region of the multigraded Betti table of $J(S \otimes_{\mathbb{Z}} \ell)$ is in one-to-one correspondence with the multigraded Betti table of $I(S \otimes_{\mathbb{Z}} \ell)$ for all fields ℓ of arbitrary characteristic. There is no analogous statement for Borel-fixed ideals in characteristic zero.
[This is a joint work with Manoj Kummini]

EMMA CONNON, Dalhousie University

When do monomial ideals have linear resolutions?

In 1990 Fröberg showed that the edge ideal of a graph has a linear resolution if and only if the complement of the graph is chordal. In this talk we will discuss the generalization of Fröberg's theorem to higher dimensions. In particular we will discuss new classes of simplicial complexes which extend the notion of a chordal graph and which give rise to a necessary condition for an ideal to have a linear resolution over any field. We will also provide a necessary and sufficient combinatorial condition for a square-free monomial ideal to have a linear resolution over fields of characteristic 2.

SUSAN COOPER, Central Michigan University

Symbolic Powers of Monomial Ideals

Central to many problems in commutative algebra is the obstacle that regular and symbolic powers of homogeneous ideals are not the same. Much work has gone into comparing these powers. In particular, conjectures of Harbourne and Huneke which relate symbolic and regular powers of ideals of fat points in projective space have gained much attention. In this talk we will explore these conjectures for monomial ideals. This is joint work with Tài Hà.

LOUIZA FOULI, New Mexico State University
Lower Bounds for the depth of powers of edge ideals of graphs

We consider a simple graph its corresponding edge ideal I in a polynomial ring R. It is well known that upper bounds for the projective dimension of R/I provide lower bounds for the first non-zero homology group of the graph's independence complex. Determining upper bounds for the projective dimension of R/I is equivalent to finding lower bounds for the depth of R/I. We discuss such bounds as well as lower bounds for the depth of higher powers of the edge ideal. This is joint work with Susan Morey.

ELENA GUARDO, University of Catania
Star configuration on generic hypersurfaces

Let F be a homogeneous polynomial in $S = \mathbb{C}[x_0, \ldots, x_n]$. Our goal is to understand a particular polynomial decomposition of F; geometrically, we wish to determine when the hypersurface defined by F in $\mathbb{P}^n$ contains a star configuration. To solve this problem, we use techniques from commutative algebra and algebraic geometry to reduce our question to computing the rank of a matrix. This is a joint work with E. Carlini and A. Van Tuyl.

BRIAN HARBOURNE, University of Nebraska-Lincoln
Advances on Recent Containment Counterexamples

It is known that the symbolic fourth power $I^{(4)}$ of every homogeneous ideal I in the homogeneous coordinate ring R of the projective plane is contained in the square I^2 of the ideal. Until recently, no examples of such an I were known where I^2 failed to contain the symbolic cube $I^{(3)}$. I will discuss the known examples and some additional recent results.

JUERGEN HERZOG, Universitaet Duisburg-Essen
On the subadditivity problem for maximal shifts in free resolutions

In this lecture I report on recent joint work with Hema Srinivasan. In many cases, including monomial ideals, subadditivity for the maximal shifts in finite graded free resolutions is expected. Some partial results will be presented.

PIOTR JEDRZEJEWICZ, Nicolaus Copernicus University
A homogeneous generalization of a theorem of Ganong and Daigle

Let A be the polynomial algebra in n variables over a field of arbitrary characteristic. We will describe all subalgebras of A, generated by n homogeneous polynomials and containing prime powers of variables. This is a joint work with Andrzej Nowicki. The motivation comes from a theorem of Ganong and Daigle for two variables in characteristic p.

JACK JEFFRIES, University of Utah
The j-Multiplicity of Monomial Ideals

The j-multiplicity was introduced by Achilles and Manaresi in 1993 as a generalization of the Hilbert-Samuel multiplicity for arbitrary ideals in a Noetherian ring. Many of the properties and algebraic applications of the Hilbert-Samuel multiplicity of $\mathfrak{m}$-primary ideals have been extended to more general classes of ideals using the j-multiplicity. In this talk we will discuss joint work with Jonathan Montaño in which we give a description of the j-multiplicity of a monomial ideal as the normalized volume of a polytopal complex.

ANTONIO MACCHIA, Università degli Studi di Bari
The Arithmetical Rank of the Edge Ideals of Graphs with Whiskers

We consider the edge ideals of large classes of graphs with whiskers and for these ideals we prove that the arithmetical rank is equal to the big height. Then we extend these results to other classes of squarefree monomial ideals, generated in any degree, proving that the same equality holds.

JEFF MERMIN, Oklahoma State University
An unsatisfying bijection

Let I be the smallest Borel-fixed ideal containing the monomial $(x_1, x_2, \cdots, x_n)$. Recently we discovered that the graded Betti numbers of I count the pointed pseudo-triangulations of a geometric configuration called the *single chain*. The connection was purely numerical, so shed no light on the combinatorial structure of either object. Now, we define bijections connecting pointed pseudo-triangulations, marked binary trees, and a basis for the resolution of I. These bijections are unsatisfying in the sense that the differential from the resolution does not appear to correspond to a natural map on pointed pseudo-triangulations.

All work is joint with Chris Francisco and Jay Schweig.

UWE NAGEL, University of Kentucky
On the Weak Lefschetz Property for Artinian Gorenstein algebras

It is an open problem whether all height three graded Artinian Gorenstein algebras have the Weak Lefschetz Property. We show that it is enough to establish this for all compressed Gorenstein algebras of odd socle degree. Then we discuss the first open case, namely algebras with Hilbert function $(1, 3, 6, 6, 3, 1)$.

This is joint work with Mats Boij, Juan Migliore, Rosa M. Miró-Roig, and Fabrizio Zanello.

AUGUSTINE O'KEEFE, University of Kentucky
Cellular resolutions of some monomial ideals

In 2009 Nagel and Reiner showed that the minimal free resolution of $\mathfrak{m}^d$, where $\mathfrak{m}$ is the maximal ideal of the polynomial ring $k[x_1, \ldots, x_n]$, is supported by a mixed subdivision of $d\Delta_n$, the d^{th} dilation of the $n-1$-dimensional simplex. In this talk we will explore monomial ideals of the form $\mathfrak{m}^d + \langle x_1^{a_1}, \ldots, x_n^{a_n} \rangle$ and show that their minimal free resolutions are supported by a deformation of the aforementioned mixed subdivision of $d\Delta_n$. As a special case, we can show that the initial ideals of Riemann-Roch monomial ideals, introduced by Manjunath and Sturmfels in 2012, have minimal cellular resolutions.

SONJA PETROVIC, Penn State / IIT
Toric algebra of hypergraphs

The edges of any uniform hypergraph parametrize a monomial algebra called the edge subring of the hypergraph. We study presentation ideals of these edge subrings, and describe their generators in terms of balanced walks on hypergraphs. Our results generalize those for the defining ideals of edge subrings of graphs, which are well-known in the commutative algebra community, and popular in the algebraic statistics community. One of the motivations for studying toric ideals of hypergraphs comes from algebraic statistics, where generators of the toric ideal give a basis for random walks on fibers of the statistical model specified by the hypergraph. Further, understanding the structure of the generators gives insight into the model geometry.

STEPHEN STURGEON, University of Kentucky
Cellular Resolution of the n-gon

Cellular Resolutions have been studied in a number of special classes. We will present a cellular resolution of the Stanley-Reisner ring of the n-gon. This is an interesting case as this ring is Gorenstein and has a non-linear resolution. Our construction shows that a resolution is supported on a self-dual polytope with integer coordinates. We can also show that this set of polytopes support a resolution of the Stanley-Reisner ring of a family of stacked polytopes.

DAVID WEHLAU, Royal Military College of Canada
Horn's Conjecture, The Littlewood-Richardson Cone and Permutations

Horn conjecture's deals with the following problem: If A and B are Hermitian matrices, how are the eigenvalues of $A+B$ constrained by the eigenvalues of A and B? In 1962, Alfred Horn conjectured a beautiful answer to this question. In the late 1990's, A. Klyachko and Knutson-Tao independently proved Horn's conjecture.

Horn's conjecture has surprising and very important connections with several areas of mathematics, including the Schubert calculus, representations of Lie Groups, and quiver theory. The space of eigenvalues of A, B and $A+B$ forms a real polyhedral cone, known as the Littlewood-Richardson Cone. Understanding the geometry of the Littlewood-Richardson Cone, yields important information.

We can describe the generating rays and many other faces of this cone in terms of inversion sets for elements of the symmetric group. I will describe this combinatorial problem, our solution and a few of the consequences.

RUSS WOODROOFE, Mississippi State University
An absence of leaves in regularity

Let G be a graph, and $I(G)$ the associated edge ideal. It is not difficult to show that there is a vertex v such that $\operatorname{reg} I(G) \leq \operatorname{reg} I(G \setminus N[v]) + 1$. In recent joint work with Tài Hà, we have shown that this vertex v can be chosen to avoid vertices of degree 1 ("leaves"). As a corollary, we get a new packing-type upper bound for the regularity of an edge ideal of a graph.