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Poisson Boundary and Low-Pass Filters

Low-pass filters associated with scaling functions for multiresolution analysis wavelets satisfy a quadrature mirror condition: $|m(\xi/2)|^2 + |m(\xi/2 + 1/2)|^2 = 1$ a.e.. Given a starting point $\xi = \xi_0$, the values $|m(\xi/2)|^2$ and $|m(\xi/2 + 1/2)|^2$ can be considered as transition probabilities for a random walk/Markov process on the torus. A better way to think of this, however, is as a random walk down a binary tree, with root ξ_0, nodes $\xi_0/2$, $\xi_0/2 + 1/2$ on the first level, and so on.

The first half of the talk will introduce some notions of a boundary for this tree and random walk, including the hyperbolic boundary, Martin boundary, and Poisson boundary. Application of these tools in the study of wavelets and analysis on fractals is relatively new, so one aim of this talk it to improve awareness of the tools among the wavelets and fractals community. The second half of the talk will note some connections between characterization theorems for scaling functions (by Hernandez and Weiss) and for low-pass filters (by Gundy) and properties of the Poisson boundary. The results hold for multiresolution analyses in any dimension with any dilation matrix, but due to time constraints will only be stated in the one-dimensional, dilation by 2 case.