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Classification of self-affine tile digit sets as product-forms

Let A be an expanding matrix on $\mathbb{R}^s$ with integral entries. A fundamental question in the fractal tiling theory is to understand the structure of the digit set $\mathcal{D} \subset \mathbb{Z}^s$ so that the integral self-affine set $T(A, \mathcal{D})$ is a translational tile on $\mathbb{R}^s$. We first show that a tile digit set in $\mathbb{Z}^s$ must be an integer tile (i.e. $\mathcal{D} \oplus \mathcal{L} = \mathbb{Z}^s$ for some discrete set $\mathcal{L}$). We completely classify such tile digit sets $\mathcal{D} \subset \mathbb{Z}$ on $\mathbb{R}^1$ by expressing the mask polynomial $P_{\mathcal{D}}$ into product of cyclotomic polynomials and putting it in the trees of cyclotomic polynomials. This allows us to combine the technique of Coven and Meyerowitz on integer tiling on $\mathbb{R}^1$ to characterize explicitly all tile digit sets $\mathcal{D} \subset \mathbb{Z}$ with $A = p^\alpha q$ (p, q distinct primes) as *modulo product-form* of some order, an advance of the previously known results for $A = p^\alpha$ and pq.