Nonlinear Partial Differential Equations and their Applications
Équations différentielles partielles non linéaires et leurs applications
(Org: **George Chen** and/et **Scott Rodney** (Cape Breton))

STEPHEN ANCO, Brock University
Symmetry analysis and exact solutions of nonlinear wave equations

Exact solutions have an important role in the study of nonlinear wave equations, particularly for understanding blow-up or dispersive behaviour, attractors, and critical dynamics, as well as for testing numerical solution methods. This talk will illustrate the application of a novel symmetry-group method to obtain explicit solutions to semilinear Schrodinger equations in multi-dimensions.

Semilinear Schrodinger equations provide a model of many interesting types of nonlinear waves, e.g. laser beams in nonlinear optics, oscillations of plasma waves, and free surface water waves. Many explicit new solutions are obtained which have interesting analytical behavior connected with blow-up and dispersion in such models. These solutions include new similarity solutions and other new group-invariant solutions, as well as new solutions that are not invariant under any symmetries of the Schrodinger equation. In contrast, standard symmetry reduction methods lead to nonlinear ODEs for which few if any explicit solutions can be derived by familiar integration methods.

LIA BRONSARD, McMaster University
Singular Limits for Thin Film Superconductors in Strong Magnetic Fields

We consider singular limits of the three-dimensional Ginzburg-Landau functional for a superconductor with thin-film geometry, in a constant external magnetic field. The superconducting domain has characteristic thickness on the scale $\epsilon > 0$, and we consider the simultaneous limit as the thickness $\epsilon \to 0$ and the Ginzburg-Landau parameter $\kappa \to \infty$. We assume that the applied field is strong (on the order of ϵ^{-1} in magnitude) in its components tangential to the film domain, and of order $\log \kappa$ in its dependence on κ. We prove that the Ginzburg-Landau energy Γ-converges to an energy associated with a two-obstacle problem, posed on the planar domain which supports the thin film. The same limit is obtained regardless of the relationship between ϵ and κ in the limit. Two illustrative examples are presented, each of which demonstrating how the curvature of the film can induce the presence of both (positively oriented) vortices and (negatively oriented) antivortices coexisting in a global minimizer of the energy. This is joint work with Stan Alama and Bernardo Galvão-Sousa.

SHAOHUA GEORGE CHEN, Cape Breton University
Global and Blowup Solutions for Quasilinear Parabolic Equations Not in Divergence Form

In this talk, we will discuss global and blowup solutions of the quasilinear parabolic equation $u_t = \alpha(x,u,\nabla u)\Delta u + f(x,u,\nabla u)$ with homogeneous Dirichlet boundary conditions. We will give sufficient conditions such that the solutions either exist globally or blow up in a finite time for any smooth initial values. In special cases, a necessary and sufficient condition for global existence is given.

SHIBING CHEN, Utoronto
Ancient solution to the generalized curve shortening flow

We prove some estimates for convex ancient solutions (the existence time for the solution starts from $-\infty$) to the generalized curve shortening flow(convex curve evolving in its normal direction with speed equal to a power of its curvature, the power is assumed to be bigger than $\frac{1}{2}$). As an application, we show that, if the convex compact ancient solution sweeps the whole space $\mathbf{R}^2$, it must be a shrinking circle. By exploiting the affine invariance of the affine curve shortening flow (when the power equals to $\frac{1}{3}$), we are also able to show that the only convex compact ancient solution must be a shrinking ellipse.

ANDRÈS CONTRERAS, Mcmaster University
Stable Vortex States in Superconductivity

We present the construction of local minimizers to the Ginzburg-Landau functional of superconductivity in the presence of an external magnetic field. We investigate the existence of stable states where the number of vortices N is far from optimal(as dictated by the energy formulation), is prescribed and blows up as the parameter epsilon, inverse of the Ginzburg-Landau parameter kappa, tends to zero. We treat the case of N as large as log ε, and a wide range of intensity of external magnetic field. This is joint work with Sylvia Serfaty.

GALIA DAFNI, Concordia University
Local Hardy spaces and applications

We review some recent results about local Hardy spaces and discuss possible applications to nonlinear PDE.

BERNARDO GALVAO-SOUSA, University of Toronto
Accelerating Fronts in Semilinear Wave Equations

I will study dynamics of interfaces in solutions of the equation $\varepsilon\Box u + \frac{1}{\varepsilon} f_\varepsilon(u) = 0$, for f_ε of the form $f_\varepsilon(u) = (u^2 - 1)(2u - \varepsilon\kappa)$, for $\kappa \in \mathbb{R}$, as well as more general, but qualitatively similar, nonlinearities. I will show that for suitable initial data, solutions exhibit interfaces that sweep out timelike hypersurfaces of mean curvature proportional to κ.

This is a joint work with Robert Jerrard (University of Toronto).

LYUDMILA KOROBENKO, University of Calgary
Hypoellipticity of Infinitely Degenerate Second Order Quasilinear Operators

The talk is concerned with regularity of weak solutions to second order infinitely degenerate elliptic equations. One of the ways to describe the regularity is in terms of the operator being subelliptic or hypoelliptic. A criteria of subellipticity for linear operators have been given by Fefferman and Phong in terms of subunit metric balls associated to the operator. In particular, it follows that an infinitely degenerate operator cannot be subelliptic. Hypoellipticity is a weaker property, and for a certain class of such operators has been recently shown by Rios, Sawyer and Wheeden in the a priori assumption that weak solutions are continuous. We use the subunit metric approach to show continuity of weak solutions to a certain class of degenerate quasilinear equations. This together with the result by Rios et al completes the proof of hypoellipticity of a class of infinitely degenerate quasilinear operators.

DUNG LE, UTSA
On the Regular Set of BMO Weak Solutions to p-Laplacian Strongly Coupled Nonregular Parabolic Systems

We will discuss the regular set of weak solutions to strongly coupled elliptic systems. We do not assume that the solutions are bounded but BMO and the ellipticity constants can be unbounded.

MING MEI, McGill University
Stability of non-monotone traveling waves for Nicholson's blowflies equations

In this talk, we consider Nicholson's blowflies equations. When the ratio of birth rate and death rate satisfies $p/d > e$, the equation losses its monotonicity, and the traveling waves are non-monotone and oscillating when the delay time r is big. The stability of such kind waves have been open and challenging as we know. In this talk, we prove that, when $e < \frac{p}{d} < e^2$, for any delay time $r > 0$, the traveling waves $\phi(x + ct)$ with $c > c_* > 0$ ($c_* > 0$ is the minimum wave speed) are asymptotically stable, when the initial perturbation is small enough; while, when $p/d \geq e^2$, we prove that these oscillating traveling waves are stable only for a small delay time $r \ll 1$, and unstable for $r \gg 1$. All these theoretical results are also confirmed numerically by some computing simulations.

This is a joint work with C.-K. Lin, C.-T. Lin and Y.-P. Lin.

KABE MOEN, University of Alabama

Regularity of solutions to degenerate p-Laplacian equations

Motivated by mappings of finite distortion, we consider degenerate p-Laplacian equations whose ellipticity condition is satisfied by the distortion tensor and the inner distortion function of such a mapping. Assuming a certain Muckenhoupt type conditions on the weight involved in the ellipticity condition, we describe the set of continuity of solutions.

DARIO MONTICELLI, Università degli Studi di Milano

Sign-changing solutions of the critical equation for the sublaplacian on the Heisenberg group

In this talk we will consider the subelliptic PDE

$$-\Delta_H u = |u|^{\frac{4}{Q-2}} u$$

on the Heisenberg group $\mathbf{H}^n$, where Δ_H is the sublaplacian operator on $\mathbf{H}^n$ and $Q = 2n + 2$ is its homogeneous dimension. The differential operator is linear, second order, degenerate elliptic, and it is hypoelliptic being the sum of squares of (smooth) vector fields satisfying the Hormander condition.

The critical growth $\frac{Q+2}{Q-2}$ in the nonlinearity of the equation is related to a loss of compactness of the continuous embeddings of suitable anisotropic Sobolev-type spaces in standard L^p spaces (on bounded domains of $\mathbf{H}^n$), which occurs at the critical exponent $p^* = \frac{2Q}{Q-2}$.

Such an equation is related for instance to problems of differential geometry on Cauchy–Riemann manifolds, such as the CR-Yamabe problem of prescribing the Tanaka-Webster scalar curvature on $\mathbf{H}^n$ under a conformal change of the contact structure that identifies its CR structure.

We will show existence of infinitely many geometrically distinct sign changing solutions of the equation on $\mathbf{H}^n$ using variational techniques, exploiting the abundant symmetries of the equation and some notions of differential geometry in order to circumvent the lack of compactness of the energy functional, that arises at the considered critical growth.

This is a joint work with P. Mastrolia (Università degli Studi di Milano)

ADAM OBERMAN, McGill Univeristy

Convergent finite difference solvers for the Monge-Ampère equation with Optimal Transportation boundary conditions

The elliptic Monge-Ampère equation is a fully nonlinear Partial Differential Equation which originated in geometric surface theory, and has been applied in dynamic meteorology, elasticity, geometric optics, image processing and image registration. Solutions can be singular, in which case standard numerical approaches fail.

We build a finite difference solver for the Monge-Ampère equation, which converges to the unique viscosity solution of the equation. Regularity results are used to select a priori between a stable, provably convergent monotone discretization and an accurate finite difference discretization. The resulting nonlinear equations are then solved by Newton's method.

Computational results in two and three dimensions validate the claims of accuracy and solution speed. A computational example is presented which demonstrates the necessity of the use of the monotone scheme near singularities.

BRENDAN PASS, University of Alberta

Multi-marginal optimal transport on a Riemannian manifold

I will discuss joint work with Young-Heon Kim on an optimal transport problem with several marginals on a Riemannian manifold, with cost function given by the average distance squared from multiple points to their barycenter. Under a standard regularity condition on the first marginal, we prove that the optimal measure is unique and concentrated on the graph of a function over the first variable, thus inducing a Monge solution. This result generalizes McCann's polar factorization theorem on manifolds from two to several marginals, in the same sense that a well known result of Gangbo and Swiech generalizes Brenier's polar factorization theorem on $\mathbb{R}^n$.

SARAH RAYNOR, Wake Forest University
A New Approach to Soliton Stability for the KdV Equation

In this work, we consider the KdV equation in the exponentially weighted spaces of Pego and Weinstein. We prove local well-posedness of the perturbation (weighted and unweighted) in the Bourgain $X^{1,b}$ space, allowing us to recreate the Pego-Weinstein result via iteration. By combining this result with the I-method, we expect ultimately to obtain soliton stability for KdV with initial data too rough to be in H^1.

SCOTT RODNEY, Cape Breton University
A Harnack Inequality for a Class of Second Order Degenerate Quasi-Linear Equations

Recent progress on the regularity of weak solutions to a class of degenerate quasi-linear second order equations with rough coefficients will be discussed. An equation in the class considered has the form

$$\mathrm{Div}\,(A(x,u,\nabla u)) = B(x,u,\nabla u)$$

where the functions A and B are assumed to satisfy specific structural conditions related to those described by J. Serrin (1964) and N. Trudinger (1967). The main focus of the talk will be issues associated to the development of a Harnack inequality for weak solutions. There are several equations of interest included in the class studied. The degenerate p-Laplacian is one such example.

ERIC SAWYER, McMaster University
Two weight norm inequalities for singular integrals in higher dimension

We discuss preliminary work on two weight inequalities for Calderon-Zygmund singular integrals in Euclidean space, with emphasis on the energy condition. This is joint work with Chun-Yen Shen and Ignacio Uriarte-Tuero.

RUNZHANG XU, Harbin Engineering University
Global well-posedness for nonlinear Schrodinger equation with combined power type nonlinearities for positive initial energy

In this talk we consider the Cauchy problem of the nonlinear Schrodinger equation with combined power type nonlinearities. By introducing a family of potential wells we obtain some sharp conditions for global existence and finite time blow up of solutions. In the frame of this work, we do not require the negative initial energy, which solves some open problems existing in the known literature.