Nonlocal Interactions in Social, Physical, and Biological Sciences
Interactions non locales en sciences sociales, physiques et biologiques
(Org: **Theodore Kolokolnikov** (Dalhouise) and/et **Michael Ward** (UBC))

TUM CHATURAPRUEK, Harvey Mudd College
Crime Modeling with Lévy Flights

We extend the Short *et al.* model of crime to incorporate biased Lévy Flights for the criminal's motion, with step-sizes distributed according to a power-law distribution. Such motion is considered to be more realistic than the biased diffusion that was originally proposed. This generalization leads to fractional Laplacians. We then investigate the effect of introducing the Lévy Flights on the formation of hot-spots using linear stability and full numerics. Joint works with Jonah Breslau, Daniel Yazdi, Theodore Kolokolnikov, and Scott McCalla.

YUXIN CHEN, Dalhousie University
Equilibrium solutions to an aggregation model subject to exogenous and Newtonian endogenous forces in 2D

We study the equilibrium solutions to an aggregation system consisting of N single-species particles and one alien particle in two-dimensional space. Starting with a discrete aggregation model subject to pairwise endogenous and exogenous forces in 2D, we derive the continuum model by introducing the continuous particle density. Throughout the study, we take the the pairwise endogenous force to be Newtonian. We show that three sets of equilibrium solutions occur under applying different exogenous force exerted by the alien particle. Additionally, we analyze the stability for the annulus-like equilibrium solution with uniform density by linear perturbation off the boundaries of the domain.

YANGHONG HUANG, Imperial College London
Self-propelled particles with quasi-Morse potential

Rich patterns are observed in self-propelled particles systems with Morse like interaction potential $U(x) = C_a e^{-|x|/\ell_a} - C_r e^{-|x|/\ell_r}$. However, the explicit forms of the observed patterns like flocks and mills are not available in higher dimension. In this talk, the potential is replaced by a quasi-Morse potentail [Carrillo *et.al*, 2013 Physca D], which consists the difference of two rescaled Bessel potentails. A few observed patterns can be obtained by solved some algebraic equations, leading to an extensive parametric study of the underlying particle system. The stability of certain patterns are also disccused.

DAVID IRON, Dalhousie
Lattice patterns in the periodic Gierer-Meinhardt system

We consider the Gierer-Meinhardt equations posed on the plane. The stability of spike solutions in which the spikes centres line up on a lattice will depend on the value of the regular part of the quasi-periodic Green's function on that lattice. The Green's function may be represented as an infinite sum, but it converges very slowly. We will show how to evaluate this Green's function quickly and determine the stability of a given lattice formation.

This is ongoing work with Dr. John Rumsey and Dr. Michael Ward.

THEODORE KOLOKOLNIKOV, Dalhousie
Vortex swarms

We investigate the dynamics of N point vortices in the plane, in the limit of large N. We consider *relative equilibria*, which are rigidly rotating lattice-like configurations of vortices. These configurations were observed in several recent experiments [Durkin and Fajans, Phys. Fluids (2000) 12, 289–293; Grzybowski *et.al* PRE (2001)64, 011603]. We show that these solutions and their stability are fully characterized via a related *aggregation model* which was recently investigated in the context of biological swarms [Fetecau *et.al.*, Nonlinearity (2011) 2681; Bertozzi *et.al.*, M3AS (2011)]. By utilizing this connection, we give explicit analytic formulae for many of the configurations that have been observed experimentally. These include configurations of vortices of equal strength; the $N+1$ configurations of N vortices of equal strength and one vortex of much higher strength; and more generally, $N+K$ configurations. We also give examples of configurations that have not been studied experimentally, including $N+2$ configurations where N vortices aggregate inside an ellipse.

CHRIS LEVY, Dalhousie University
Dynamics and Stability of a 3D Model of Cell Signal Transduction with Delay

We consider a 3D model of cell signal transduction with delay. In this model, the deactivation of signalling proteins occur throughout the cytosol and activation is localized to specific sites in the cell. We use matched asymptotic expansions to construct the dynamic solutions of signalling protein concentrations. The result of the asymptotic analysis is a system of delay differential equations (DDEs). This reduced DDE system is compared to numerical simulations of the full 3D system with delay. There are delay values which give rise to sustained oscillations. We implement the method of constrained coordinates numerically to improve the asymptotic results in this case.

ALAN LINDSAY, Heriot-Watt University
The Stability and Evolution of Curved Domain Arising From One Dimensional Localized Patterns

In many pattern forming systems, narrow two dimensional domains can arise whose cross sections are roughly one dimensional localized solutions. This talk will present an investigation of this phenomenon for the variational Swift-Hohenberg equation. Stability of straight line solutions is analyzed, leading to criteria for either curve buckling or curve disintegration. A high order matched asymptotic expansion reveals a two-term expression for the geometric motion of curved domains which includes both elastic and surface diffusion-type regularizations of curve motion. This leads to novel equilibrium curves and space-filling pattern proliferation. A key ingredient in the generation of the labyrinthine patterns formed, is the non-local interaction of the curved domain with its distal segments. Numerical tests are used to confirm and illustrate these phenomena.

RYAN LUKEMAN, St. Francis Xavier University
Transition Dynamics in Collective Animal Motion

One striking feature of collective motion in animal groups is a high degree of alignment among individuals, generating polarized motion. When order is lost, the dynamics process of reorganization provides information about both the nature of the interaction rules governing the motion of individuals, and how these rules affect the functioning of the collective.

In this talk, I describe a dataset of trajectories of collectively swimming surf scoters (an aquatic duck) during transitions between order and disorder. The data is used to determine signatures of the transition, and what inter-individual interactions permit efficient recovery of order in these groups. Results suggest an adaptive explanation for the individual rules adopted by group members: those which permit flexibility and efficient recovery of order from a perturbation.

ALAN MACKEY, UCLA
Two-species Particle Aggregation and Co-dimension One Solutions

Systems of pairwise-interacting particles model a cornucopia of physical systems, from insect swarms and bacterial colonies to nanoparticle self-assembly. We study a continuum model for two-species particle interaction in $\mathbb{R}^2$, and apply linear stability analysis of concentric ring steady states to characterize the steady state patterns and instabilities which form. Conditions for linear well-posedness are determined and these results are compared to simulations of the discrete particle dynamics, showing predictive power of the linear theory.

XIAOFENG REN, George Washington University

A double bubble solution in a ternary system with inhibitory long range interaction

We consider a ternary system of three constituents, a model motivated by the triblock copolymer theory. The free energy of the system consists of two parts: an interfacial energy coming from the boundaries separating the three constituents, and a longer range interaction energy that functions as an inhibitor to limit micro domain growth. We show that a perturbed double bubble exists as a stable solution of the system. Each bubble is occupied by one constituent. The third constituent fills the complement of the double bubble. Two techniques are developed. First one defines restricted classes of perturbed double bubbles. Each perturbed double bubble in a restricted class is obtained from a standard double bubble by a special perturbation. The second technique is the use of the so called internal variables. The advantage of the internal variables is that they are only subject to linear constraints, and perturbed double bubbles in each restricted class represented by internal variables are elements of a Hilbert space. A local minimizer of the free energy in each restricted class is found as a fixed point of a nonlinear equation. This perturbed double bubble satisfies three of the four equations for critical points of the free energy. The unsolved equation is the 120 degree angle condition at triple junction points. Perform another minimization among the local minimizers from all restricted classes. A minimum of minimizers emerges and solves all the equations for critical points.

W ABOU SALEM, University of Saskatchewan

Semi-Relativistic Schroedinger-Poisson System of Equations with Long-Range Interactions

The evolution of the density matrix of interacting many-body quantum particles in the mean-field limit is given by the Hartree-von Neumann equation. Using the properties of the density matrix, this is equivalent to a system of infinitely coupled nonlinear PDEs, the Schroedinger-Poisson system of equations. The semi-relativistic Schroedinger-Poisson system of equations describes the mean-field dynamics of interacting quantum particles with very high velocities, such as in plasmas. I discuss the derivation of the semi-relativistic Schroedinger-Poisson system of equations with long-range interactions and its global well-posedness in appropriate Sobolev spaces. I also describe the asymptotics of the solution as the mass of the particles tends to zero and to infinity, respectively.

JONATHAN SHERRATT, Heriot-Watt University

A Nonlocal Model for Cancer Invasion

Adhesion of cells to one another and their environment is an important regulator of many biological processes, but has proved difficult to incorporate into continuum mathematical models. I will describe a new approach to the mathematical modelling of adhesion in cell populations, based on an integro-partial differential equation for cell density, in which the integral represents the sensing by cells of their local environment. This enables an effective representation of cell-cell adhesion, as well as random cell movement, and cell proliferation. I will show how this modelling approach can be applied to cancer growth. In this context, the model is capable of supporting both benign (non-invasive) and invasive growth, according to the relative strengths of cell-cell and cell-matrix adhesion. I will go on to describe the use of the model to investigate the criticality of matrix heterogeneity in shaping invasion, making the testable prediction that highly heterogeneous extracellular matrix can result in a "fingering" of the tumour front, which is a hallmark of invasive cancers.