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**Krieger-Nelson Prize Lecture**  
**Conférence - Prix Krieger-Nelson**

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*Average distributions for elliptic curves*

Let  $E$  be an elliptic curve over  $\mathbf{Q}$ . There are many open conjectures about the distribution of local invariants associated with the reductions of  $E$  modulo  $p$  as  $p$  varies over the primes. Perhaps the most famous examples are the conjectures of Lang and Trotter (1976) and Koblitz (1988). In order to gain evidence for the conjectures, it is natural to consider the average distribution over some families of elliptic curves. We explain how the average results fit the conjectural asymptotics, in terms of the order of magnitude, but also in terms of the precise constants associated to each given conjecture, giving evidence for the probabilistic models defined in terms of local probabilities. More recently, applying those average techniques to different distribution questions, as counting the frequency of occurrence of a given abelian group appearing as the group of points of elliptic curves over finite fields, we found that the resulting average distribution is also governed by the Cohen-Lenstra Heuristics, which predict that random abelian groups occur with probability weighted by the number of elements of their automorphism group.