Interactions Between Algebraic Geometry and Commutative Algebra
Intéractions entre la géométrie algébrique et l'algèbre commutative
(Org: **Susan Cooper** (Central Michigan) and/et **Sean Sather-Wagstaff** (North Dakota State))

KRISTEN BECK, University of Arizona
Depth and dimension for high syzygies

This talk will focus on the asymptotic behavior of two invariants, depth and dimension, for syzygies of a finitely generated module of infinite projective dimension over a commutative local noetherian ring. It is already known, by work of Okiyama (1991), that the depths of such syzygies eventually stabilize to that of the ring. This result is natural, given the intimate connection between depth and free resolutions. However, formulating a similar statement for dimension is not as straightforward. In this talk, we will give necessary conditions for the stabilization of dimensions of syzygy modules. In particular, we will show that all high syzygies of a module with eventually non-decreasing Betti numbers over an equidimensional ring have dimension equal to the dimension of the ring.

JENNIFER BIERMANN, Lakehead University
Balanced vertex decomposable simplicial complexes and their h-vectors

Given any finite simplicial complex Δ, I will discuss how to construct a new simplicial complex Δ_χ that is balanced and vertex decomposable and whose h-vector is the f-vector of the original complex. This construction generalizes the "whiskering" construction of Villarreal, and Cook and Nagel. I will also discuss how to reverse this process in the special case of the independence complex of a chordal graph.

ENRICO CARLINI, Politecnico di Torino
Determining the Waring rank: special cases

The Waring rank of an homogeneous degree d polynomial $F(x_1,\ldots,x_n)$ is the minimal s such that we can write

$$F = L_1^d + \ldots + L_s^d,$$

where the L_i are linear forms. As a matter of fact, there is no effective algorithm to compute the Waring rank, $\mathrm{rk}(F)$, of a given polynomial. Thus we will show the few cases in which $\mathrm{rk}(F)$ is explicitly known. Namely, if F is a degree two form (classically known) or if F is a monomial or a sum of coprime monomials. This is based on joint work with M.V.Catalisano and A.V.Geramita.

ANNIKA DENKERT, University of Nebraska - Lincoln
Resurgence and related questions for two intersecting lines in $\mathbb{P}^2$

Given an ideal I in a polynomial ring over a field, we can define the resurgence of I as the supremum over all ratios m/r such that the m-th symbolic power $I^{(m)}$ is not contained in the r-th ordinary power I^r. We will exhibit some results for the resurgence and related questions in the case that I is the ideal defined by $2n+1$ distinct points in $\mathbb{P}^2$, where n points lie on a line L_1, n points lie on line L_2, and one point is at the intersection of L_1 with L_2.

SARA FARIDI, Dalhousie University
Monomial resolutions supported by simplicial trees

This talk is about resolutions of monomial ideals. Given a monomial ideal, Bayer, Peeva and Sturmfels provided a criterion for when a simplicial complex labeled by the generators of the ideal would support a free resolution of the ideal, expanding earlier work of Taylor. In this talk we discuss what happens when the supporting simplicial complex is a tree.

CHRIS FRANCISCO, Oklahoma State University
Borel ideals and connections to discrete geometry

We'll discuss Borel ideals and their generalizations along with a surprising connection to a counting problem in discrete geometry. This is joint work with Jeff Mermin and Jay Schweig.

NATHAN GRIEVE, Queen's University
Cup-product problems on an abelian variety

This is a report on work in progress concerning certain cup-product problems of line bundles on an abelian variety. At present the state of affairs is as follows. (i) Such problems are not, in general, independent of the numerical classes of the given line bundles; (ii) After scaling every non-trivial problem results in a surjective map; (iii) Variants of the above hold for (higher rank) vector bundles and translations thereof. As an application we deduce that, after scaling, certain (higher) skew-Pontrjagin products are globally generated.

BRIAN HARBOURNE, University of Nebraska-Lincoln
Containments of symbolic powers in ordinary powers

We discuss joint work with Cristiano Bocci and Susan Cooper verifying cases of refined containment conjectures of the speaker with Craig Huneke. Given a homogeneous ideal I in a polynomial ring, the question is which symbolic powers of I are contained in a given ordinary power of I. The refinements involve multiplying the powers of I by powers of the irrelevant ideal.

ANDREW HOEFEL, Queen's University
Powers of edge ideals with linear resolutions

Let $I(G)$ be the edge ideal of a simple graph G and let F_k be the set of simple graphs G for which $I(G)^d$ has a linear resolution for all $d \geq k$. Although Herzog, Hibi and Zheng showed that F_1 is the set of chordal graphs, combinatorial classifications of F_k for $k \geq 2$ remain to be found. Nevo's family of claw and four cycle free graphs may be a subset of F_2 since their second powers have linear resolutions, but it is not known whether the higher powers of these graphs also have linear resolutions. I will be talking about combinatorial techniques for showing higher powers of edge ideals have linear resolutions in an effort to identify subsets of the F_k.

MIKE JANSSEN, University of Nebraska-Lincoln
Ideals of almost collinear points in $\mathbb{P}^2$

In recent years, much work has been done on the question of comparing symbolic and ordinary powers of an ideal I in a Noetherian ring. In particular, several authors have asked: for which m and r is the symbolic power $I^{(m)}$ contained in the ordinary power I^r? Recent results of Ein, Lazarsfeld, and Smith; Hochster and Huneke; Bocci and Harbourne; and Harbourne and Huneke, address various aspects of this problem. Our results answer the containment question in a geometric setting and include the computation of various invariants of the ideal in question.

MELISSA LINDSEY, Indiana Wesleyan University
Using lex-plus-powers ideals to maximize global Betti numbers

We investigate the global Betti numbers of homogeneous ideals in a polynomial ring that contain a monomial complete intersection. Given a fixed monomial complete intersection in a polynomial ring we determine the maximum first and last Betti numbers for homogeneous ideals that contain the complete intersection. In the case of a polynomial ring in three variables we also determine the maximum second Betti number. In this setting, we know from a result of Mermin-Murai that the global

Betti numbers will be maximized by a lex-plus-powers ideal. The results also make use of the structure of the Hilbert function of a complete intersection as well as the concept of e-monomials.

UWE NAGEL, University of Kentucky
Gorenstein algebras presented by quadrics

We discuss restrictions on the Hilbert function of standard graded Gorenstein algebras with only quadratic relations. Furthermore, we pose some intriguing conjectures and provide evidence for them by proving them in some cases using a number of different techniques, including liaison theory and generic initial ideals.

LUKE OEDING, UC Berkeley
The trifocal variety

In Computer Vision the multi-view variety is constructed by considering several cameras in general position in space, all focused on the same point. Given lines in all camera planes, how can I tell if the cameras were all looking at the same point? This question can be answered by finding implicit defining equations for the multi-view variety. We may also be interested in the algebraic problem to find the minimal generators of the defining ideal of the multi-view variety. The case of 3 cameras, the trifocal variety, is already interesting. In this talk I will explain our use of symbolic and numerical computations aided by Representation Theory and Numerical Algebraic Geometry to study the ideal of the trifocal variety. Our work builds on the work of others (such as Hartley-Zisserman, Alzati-Tortora and Papadopoulo-Faugeras) who have already considered this problem set-theoretically. This is joint work with Chris Aholt (Washington).

MEGAN PATNOTT, University of Notre Dame
The h-vectors of arithmetically Gorenstein sets of points on a general sextic surface in $\mathbb{P}^3$

We give the possible h-vectors of arithmetically Gorenstein sets of points on a general sextic surface in $\mathbb{P}^3$ and show the existence of such a set for each possible h-vector. We also give some partial results for general surfaces of higher degree and for the possible graded Betti numbers of arithmetically Gorenstein sets of points on a general quintic surface. Our methods are centered on the vector bundle techniques developed by several people, notably Chiantini and Faenzi, together with extensive use of liaison theory.

CHRIS PETERSON, Colorado State University
Linear algebra and the Tangent Bundle

This talk will discuss several themes relating constructions in Linear Algebra to constructions in Commutative Algebra/Algebraic Geometry. The approach is homological in nature and lends itself to concrete algorithms that may be implemented in both a symbolic setting and a numeric setting. This is joint work with David Eklund.

SANDRA SPIROFF, University of Mississippi
The Vanishing of Invariants on Complete Intersections

We discuss some invariants for a pair of modules over a complete intersection ring, with special focus on the graded case. In particular, we study a new invariant when the ring has only isolated singularity at the irrelevant maximal ideal or when the tensor product of the modules has finite length. We show that it shares many of the same properties as Hochster's original theta invariant, defined for hypersurfaces. In particular, it vanishes if and only if the dimension inequality is satisfied.

STEPHEN STURGEON, University of Kentucky
Combinatorics of Boij-Soderberg Theory

Boij-Söderberg theory gives a unique decomposition of the betti tables of modules. This decomposition produces certain pure diagrams along with integer coefficients. Considering certain classes of modules we can give an interpretation to these integer sequences. Our primary results give the decomposition of all ideals with linear resolutions and a certain class of Gorenstein ideals.