
Geometry and Topology of Lie Transformation Groups
Géométrie et topologie des groupes de transformation de Lie
(Org: **Lisa Jeffrey** (Toronto) and/et **Liviu Mare** (Regina))

TOM BAIRD, Memorial University of Newfoundland
GKM-sheaves and equivariant cohomology

Let T be a compact torus. Goresky, Kottwitz and Macpherson showed that for a large and interesting class of T -manifolds M , the equivariant cohomology ring $H_T^*(M)$ can be encoded in a graph, now called a GKM-graph, with vertices corresponding to the fixed points M^T and edges labeled by characters of T .

In this lecture, we explain how the GKM construction can be generalized to any compact T -manifold. This generalization gives rise to new mathematical objects: GKM-hypergraphs and GKM-sheaves. If time permits, we will show how these methods were used to resolve a conjecture concerning the moduli space of at connections over a non-orientable surface.

JONATHAN FISHER, University of Toronto
The Kirwan method in hyperkähler geometry

Following ideas of Atiyah and Bott, Kirwan studied symplectic reductions $M//K$ using K -equivariant Morse theory on M . In particular, she proved that the Kirwan map, a natural map from the K -equivariant cohomology of M to the ordinary cohomology of the reduced space $M//K$, is surjective. In hyperkähler geometry, there is a quotient construction analogous to symplectic reduction, but it is not known in general whether the hyperkähler Kirwan map is surjective. I will describe recent work in which we develop the requisite Morse theory, at least when K is abelian. This gives surjectivity results for all hypertoric varieties, as well as certain Nakajima quiver varieties and some related quotients. This is joint work with Lisa Jeffrey.

MATTHIAS FRANZ, University of Western Ontario
Equivariant cohomology, syzygies and orbit structure

The GKM method is a powerful way to compute the equivariant (and ordinary) cohomology of many spaces with torus actions. So far it has only been applied to so-called equivariantly formal T -spaces, for example to compact Hamiltonian T -manifolds. In this talk I will explain that the GKM method is valid for a much larger class of T -spaces. Our result is based on a new interpretation of a sequence originally due to Atiyah and Bredon, and involves the notion of syzygies as used in commutative algebra. I will also present a surprising relation between the GKM description and the equivariant Poincaré pairing.

This is joint work with Chris Allday and Volker Puppe.

OLIVER GOERTSCHES, Universität Hamburg
Equivariant cohomology of transverse actions and K -contact manifolds

Molino showed that a Riemannian foliation on a simply-connected complete Riemannian manifold (or more generally a Killing foliation) admits a natural transverse action of a finite-dimensional Abelian Lie algebra by transverse vector fields, whose orbits are exactly the leaf closures. We will discuss equivariant cohomology for such actions, and see that it behaves in many respects similar to ordinary equivariant cohomology of a torus action: for example, there is a Borel-type localization theorem, in which the role of the fixed points is played by the closed leaves of the foliation.

We will apply this theory to the characteristic foliation of a K -contact manifold in order to understand the space of closed Reeb orbits. For example, we give a new proof of the fact that on a $2n + 1$ -dimensional K -contact manifold M there are at least $n + 1$ closed Reeb orbits, and we show that equality holds if and only if M is a real cohomology sphere.

(Based on joint work with Hiraku Nozawa and Dirk Töben)

JACQUES HURTUBISE, McGill University

Flat bundles and Grassmann framings

When considering flat unitary bundles on a punctured Riemann surface, it is often convenient to have a space that includes all possible holonomies around the punctures; such a space is provided by the extended moduli space of Jeffreys. On the other hand, there are certain inconveniences, in particular no clear link to complex geometry via a Narasimhan-Seshadri type theorem. It turns out that the situation can be remedied quite nicely by considering bundles with framings taking values in a Grassmannian. Analogs for general structure groups, and in particular links with recent work of Martens and Thaddeus, will also be discussed. (joint work with U. Bhosle and I. Biswas)

DEREK KREPSKI, McMaster University

On symplectic toric orbifolds as quotients of a finite group action

We discuss conditions under which an orbifold presented as the orbit space of a smooth action of a compact Lie group (acting almost freely on a smooth manifold) is equivalent (as a stack) to the orbit space of a finite group action. Applications to symplectic toric orbifolds (symplectic Deligne-Mumford stacks) are then considered.

JOCHEN KUTTLER, University of Alberta

Tensors of bounded rank are defined in bounded degree

The notion of tensor (border) rank of elements of the p -fold tensor product of a vector space with itself appears naturally in various areas like algebraic statistics, algebraic geometry, or complexity theory. The variety of tensors of a given border rank is relevant in these contexts. In the elementary case of $p = 2$, these varieties are of course nothing but the rank varieties of square matrices, and for rank k their ideals have generators in degree $k + 1$ (the $(k + 1) \times (k + 1)$ subdeterminants), irrespective of the size of matrices.

In this talk I will present a qualitative generalization of this result. Using symmetries of these rank varieties we approach the case of border rank k for all p simultaneously and show that the rank k -variety is generated in a degree d independent of p .

This is joint work with Jan Draisma.

GREGORY D. LANDWEBER, Bard College

Moduli spaces for off-shell supersymmetry

In physics, an on-shell representation of the Poincaré group is a space of sections over a covector orbit. An off-shell representation is, in contrast, an action of the Poincaré group on functions over the entire covector space. Extending to the super Poincaré group, on-shell representations can be classified via a supersymmetric extension of Wigner's method. However, there is no known classification of supersymmetric off-shell representations.

Restricting to the super Poincaré group for one spacetime dimension, off-shell representations correspond to filtered Clifford supermodules. We introduce a notion of equivalence between off-shell representations of supersymmetry and use it to construct moduli spaces of such representations. These moduli spaces are quotients of certain flag manifolds on spin representations by the actions of the corresponding spin groups, and we will present the low dimensional examples. We will also discuss the corresponding problem for compact Lie groups, replacing covector orbits with coadjoint orbits.

TRAVIS LI, University of Toronto

Construction of symplectic groupoids

The notion of symplectic groupoids was introduced in the 80's to study Poisson geometry. While this program has been successful, there are relatively few explicit examples. I will introduce a new method of constructing Lie groupoids as co-products of manifolds. We then apply this method to construct the symplectic groupoids of a class of Poisson manifolds known as log symplectic manifolds. This is joint work with Marco Gualtieri.

DAVID LI-BLAND, University of Toronto
Colored moduli spaces of flat connections

Suppose \mathfrak{g} is a Lie algebra carrying an invariant bilinear form. If Σ is an oriented surface, then Atiyah and Bott famously showed that the moduli space of flat \mathfrak{g} connections over Σ carries a symplectic form. In this talk we will show how to construct this moduli space using finite dimensional techniques. We will be particularly interested in surfaces with boundary (and corners), and will want to color each boundary component with coisotropic subalgebras of \mathfrak{g} . From this perspective, interesting algebraic structures on \mathfrak{g} (for instance a decomposition as the double of a Lie bialgebra) will reveal themselves as algebraic structures on the moduli spaces (for instance, the double symplectic groupoid structure integrating the double of a Lie bialgebra). This is work in progress with Pavol Severa.

LIVIU MARE, University of Regina
Nonabelian GKM Theory

This is a report on joint work with Oliver Goertsches. A theorem of Goresky, Kottwitz, and MacPherson (GKM) describes the equivariant cohomology ring for certain actions of tori on compact manifolds in terms of points whose stabilizers have codimension at most 1. I will discuss a generalization of this result to actions of compact, possibly nonabelian, Lie groups. The corresponding equivariant cohomology ring is this time determined by points whose stabilizers have corank at most 1. Like in the usual GKM theory, one can encode the equivariant cohomology ring into a certain labeled graph, which this time has some special features.

LEONARDO MIHALCEA, Virginia Tech University
Curve neighborhoods of Schubert varieties

If X is a Schubert variety in a flag manifold G/P , its curve neighborhood $X(d)$ is defined to be the union of the rational curves of degree d passing through X . It turns out that $X(d)$ is also a Schubert variety, and I will explain how to identify it explicitly in terms of the combinatorics of the Weyl group and of the associated (nil-)Hecke product. If time remains, I will also show how this yields a new, natural proof of the Chevalley formula in the quantum cohomology of flag manifolds. This is joint work with A. Buch and uses previous results joint with A. Buch, P.E. Chaput and N. Perrin.

PETER QUAST, Universität Augsburg
The 'equator' of a symmetric space

In 1988 Chen and Nagano introduced centrioles in compact symmetric spaces, which generalize equators in spheres. Centrioles are connected components of the set of midpoints between two 'antipodal' points. In this talk we discuss some beautiful geometric properties of centrioles and we sketch applications to homotopy theory.

This talk is partially based on joint work with A.-L. Mare and on joint work with M. S. Tanaka.

STEVEN RAYAN, University of Toronto
Constructions of co-Higgs bundles in higher dimensions

I will outline a couple of constructions of co-Higgs bundles, which are Higgs bundles whose Higgs fields take values in the tangent bundle. One reason why these objects are interesting is that they are precisely the generalized holomorphic bundles on an ordinary complex manifold considered as a generalized complex one.

One method produces a co-Higgs bundle on any complex manifold; in a sense, this is the canonical co-Higgs bundle. The other is specifically for the projective plane. Recall that one of the earliest constructions of interesting (i.e. indecomposable) rank-2 vector bundles on a complex surface was Schwarzenberger's construction of a vector bundle on the projective plane from a double cover. I will breathe new life into this idea by showing that this bundle carries a natural $\mathcal{O}(1)$ -valued Higgs field, which can be pushed to a T -valued Higgs field on \mathbb{P}^2 . For both examples, we will discuss their stability and deformation theory.

KATHLEEN SMITH, University of Toronto

Connectivity and Convexity Properties of the Moment Map for Group Actions on Hilbert Manifolds

Thirty years ago a landmark convexity result was obtained by Atiyah (and independently Guillemin and Sternberg): the image of the moment map for a torus action on a compact symplectic manifold is a convex polyhedron.

In this talk we present an infinite-dimensional analogue to this result. In particular, we will examine Hilbert manifolds equipped with a strongly symplectic structure and a finite-dimensional group action preserving the symplectic structure. We discuss connectedness of a generic set of regular level sets of the moment map in this generality. This is used to prove convexity of the image of the moment map.

JULIANNA TYMOCZKO, Smith College

Combinatorial tools for cohomological calculations

GKM theory is the name often given to a combinatorial tool for computing torus-equivariant cohomology of suitably well-behaved varieties. We describe an extension of GKM theory, which we call poset pinball, that allows us to calculate S^1 -equivariant cohomology for a much larger class of varieties. We finish with several examples and conjectures. Much of this work is joint with Megumi Harada (McMaster University).

CATALIN ZARA, UMass Boston

Balanced Fiber Bundles and GKM Theory

Let T be a torus and B a compact T -manifold. Goresky, Kottwitz, and MacPherson showed that if B is (what was subsequently called) a GKM manifold, then there exists a simple combinatorial description of the equivariant cohomology ring $H_T^*(B)$ as a subring of $H_T^*(B^T)$. We discuss an analogue of this result for T -equivariant fiber bundles $\pi: M \rightarrow B$. We show that there is a combinatorial description of $H_T^*(M)$ as a subring of $H_T^*(\pi^{-1}(B^T))$. Using this result we obtain fiber bundle analogues of GKM theory for homogeneous spaces. This is joint work with Victor Guillemin (MIT) and Silvia Sabatini (EPFL).