Cluster Algebras and Related Topics
Algèbres amassées et sujets reliés
(Org: **Ralf Schiffler** (Connecticut) and/et **Hugh Thomas** (UNB))

IBRAHIM ASSEM, Université de Sherbrooke
On the first Hochschild cohomology group of a cluster-tilted algebra

This is a joint work with Ralf Schiffler and Maria Julia Redondo. Given a cluster-tilted k-algebra B, we study its first Hochschild cohomology group $HH^1(B)$ with coefficients in the B-B bimodule B. If C is a tilted algebra such that B is the relation-extension of C, then we show that if C is constrained, or else if B is tame, then $HH^1(B)$ is isomorphic, as a k-vector space, to the direct sum of $HH^1(C)$ with $k^{n_{B,C}}$ where $n_{B,C}$ is an invariant linking the bound quivers of B and C. In the representation-finite case, $HH^1(B)$ can be read just by looking at the quiver of B.

JUAN CARLOS BUSTAMANTE, Université de Sherbrooke
Hochschild cohomology and the derived class of m-cluster tilted algebras of type $\mathbb{A}$

Joint work with V. Gubitosi, from Sherbrooke, see http://arxiv.org/abs/1201.4182

For a given integer m and a hereditary algebra H, the m-cluster category of H, $\mathcal{C}_m(H)$ is obtained from the derived category $\mathcal{D}(H)$ by identifying the Auslander-Reiten translation with the m^{th} power of the shift $[1]^m := [m]$. In $\mathcal{C}_m(H)$ there are cluster tilting objects, whose endomorphisms algebras are called m-cluster tilted algebras.

The aim of this work is to classify the algebras that are derived equivalent to m-cluster tilted algebras of type $\mathbb{A}$. The first result states that a connected algebra $A = kQ/I$ is derived equivalent to an m-cluster tilted algebra of type $\mathbb{A}$ if and only of it is gentle, having exactly $|Q_1| - |Q_0| + 1$ oriented cycles of length $m + 2$ each of which has full relations. We then prove:

Theorem: Let $A = kQ/I$ and $A' = k'Q'/I'$ be connected algebras derived equivalent to m-cluster tilted algebras of type $\mathbb{A}$. Then (among others) the following conditions are equivalent.

1. A and A' are derived equivalent,
2. A and A' are tilting-cotilting equivalent,
3. $\mathrm{HH}^*(A) \simeq \mathrm{HH}^*(A)$ and $K_0(A) \simeq K_0(A')$
4. $\pi_1(Q, I) \simeq \pi_1(Q', I')$ and $|Q_0| = |Q'_0|$.

Our approach differs from previous works on related topics in the fact that we use the Hochschild cohomology ring as a derived invariant. We shall discuss about the proof and derive some consequences, among which we recover previous known results.

LUCAS DAVID-ROESLER, University of Connecticut
Representation theory using surfaces

We introduce a class of finite dimensional gentle algebras, surface algebras, coming from certain partially triangulated surfaces. The construction of these algebras is motivated by the combinatorial relationship between iterated tilted algebras of Dynkin type and the cluster-tilted algebras of Dynkin type given admissible cuts of quivers. We will discuss some of the ways we can study the representation theory of these algebras using only data coming from the surface.

MICHAEL GEKHTMAN, University of Notre Dame
Cremmer-Gervais Cluster Algebras

I will report on an ongoing joint project with M. Shapiro and A. Vainshtein devoted to a conjectural correspondence between classes in the Belavin-Drinfeld classification of of Poisson-Lie structures on a simple Lie groups and cluster algebra structures in the ring of regular functions on the group. I will concentrate on the case associated with the Cremmer-Gervais r-matrix, that is the farthest away from the standard Poisson-Lie structure in SL(n) and describe the corresponding cluster algebra.

MAX GLICK, University of Michigan
Singularity confinement for the pentagram map

The pentagram map, introduced by R. Schwartz, is a birational map on the configuration space of polygons in the projective plane. We study the singularities of the iterates of the pentagram map. We show that a typical singularity disappears after a finite number of iterations, a confinement phenomenon first discovered by Schwartz. We provide a method to bypass such a singular patch by directly constructing the first subsequent iterate that is well defined on the singular locus under consideration. The key ingredient of this construction is the notion of a decorated (twisted) polygon, and the extension of the pentagram map to the corresponding decorated configuration space.

KIYOSHI IGUSA, Brandeis University
Distinguished triangles in continuous cluster categories.

Cluster categories are triangulated categories which categorify cluster algebras. We give recognition principles for distinguished triangles in the continuous cluster category using continuous deformations of triangles and show that these also give recognition principles for distinguished triangles in the standard cluster categories of type A_n and A_∞. I will begin with a brief review of the continuous Frobenius category and end with explicit formulas for which mapping make $A \to B \to C \to A[1]$ into a distinguished triangle. This is joint work with Gordana Todorov from Northeastern University.

KYUNGYONG LEE, Wayne State University
Positivity in rank 2 cluster algebras

We present a canonical basis for the cluster algebra associated to any skew-symmetrizable 2×2 integer matrix. This is joint work with Li Li, Paul Sherman and Andrei Zelevinsky.

GREGG MUSIKER, University of Minnesota
Graph theoretical formulas for certain periodic quivers

We present work in progress on combinatorial formulas for cluster variables arising from periodic cluster algebras. Here, we mean periodic in the sense of Fordy and Marsh. This includes formulas for cluster variables with principal coefficients associated to the Gale-Robinson sequence and the Aztec Diamond quiver.

DYLAN RUPEL, University of Oregon
Quantum Cluster Characters via Hall-Ringel Algebras

It was recognized in the original papers of Caldero-Chapoton and Caldero-Keller that the multiplication formulas for cluster characters resemble the multiplication in the dual Hall-Ringel algebra. In this talk we will realize the quantum cluster character via a natural algebra homomorphism from the Hall-Ringel algebra to quasi-commuting Laurent polynomials. This is joint work with Arkady Berenstein.

VASILISA SHRAMCHENKO, University of Sherbrooke
Cluster automorphisms

We introduce the notion of cluster automorphism of a given cluster algebra as a $\mathbb{Z}$-automorphism of the cluster algebra that sends a cluster to another and commutes with mutations. We study the group of cluster automorphisms in detail for acyclic cluster algebras and cluster algebras from surfaces, and we compute this group explicitly for the Dynkin and Euclidean types.

GORDANA TODOROV, Northeastern University
Morphisms detemined by objects.

Abstract: The notion of Morphism Determined by Object was introduced by Maurice Auslander in order to construct and classify morphisms, mostly in module categories. A recent renewed interest in this notion is coming from C. M. Ringel for module categories and also from H. Krause, who extended and studied this notion in triangulated categories. In this talk, I will discuss functorial interpretation of the morphisms determined by objects in terms of the socle of the cokernel functor induced by the given morphism. I will give applications in cluster categories and other related categories.

ADAM-CHRISTIAN VAN ROOSMALEN, University of Regina
Cluster categories associated to new hereditary categories

Given a finite quiver, one can associate a cluster category by considering orbits of the bounded derived category of finite dimensional representations. In this talk, we want to replace the original quiver by a suitable small category such that the orbit construction still makes sense, thus obtaining new examples of 2-Calabi-Yau categories with cluster tilting subcategories. We will consider some examples where one can use combinatorics to describe the cluster tilting subcategories, as is done by Holm and Jørgensen in the case of the infinite Dynkin quiver A_∞ using triangulations of the ∞-gon.