
MURRAY BREMNER, University of Saskatchewan

The fundamental invariants of $3 \times 3 \times 3$ arrays

We use computer algebra to determine explicitly the three fundamental invariants of a $3 \times 3 \times 3$ array over \mathbb{C} as polynomials in the 27 variables x_{ijk} for $1 \leq i, j, k \leq 3$. By the work of Vinberg on θ -groups, it is known that these invariants are homogeneous polynomials of degrees 6, 9 and 12 with respectively 1152, 9216 and 209061 terms. These three polynomials freely generate the algebra of invariants for $\mathfrak{sl}(3, \mathbb{C})^3$ acting irreducibly on its natural representation $(\mathbb{C}^3)^{\otimes 3}$. We find compact expressions for these invariants in terms of the orbits of the finite group $(S_3 \times S_3 \times S_3) \rtimes S_3$ acting on monomials of weight zero. It remains an open problem to express the hyperdeterminant of degree 36 (in the sense of Gelfand et al.) in terms of these fundamental invariants. (This is joint work with Jiexiong Hu.)