RENATE SCHEIDLER, University of Calgary

Cubic Function Field Tabulation and 3-Ranks of Hyperelliptic Curves

We present an algorithm for tabulating all cubic function fields of square-free discriminant $D(x) \in \mathbb{F}_q(x)$ up to a given discriminant degree bound B so that the hyperelliptic curve $y^2 = -3D(x)$ has only one infinite place. Our method is an extension of Belabas' technique for tabulating cubic number fields and requires $O(B^4 q^B)$ operations in $\mathbb{F}_q$ as $B \to \infty$. The main ingredient is a function field analogue of the Davenport-Heilbronn correspondence between triples of $\mathbb{F}_q(x)$-conjugate cubic function fields and certain equivalence classes of binary cubic forms over $\mathbb{F}_q(x)$, described via reduced representatives.

Our method additionally finds for any $r \in \mathbb{Z}^{\geq 0}$ all hyperelliptic curves $y^2 = -3D(x)$ whose class group has 3-rank r. For $q \equiv -1 \pmod 3$, our numerical data largely supports the predicted heuristics of Friedman-Washington and partial results on the distribution of the counts of such curves due to Ellenberg-Venkatesh-Westerland. For $q \equiv 1 \pmod 3$, our data seems to agree with a result due to Achter as well as recent conjectures due to Garton that incorporate into the Friedmann-Washington heuristics a correction factor first proposed by Malle for the number field scenario.