Homotopy Theory
Théorie de l'homotopie
(Org: **Kristine Bauer** (Calgary) and/et **Marcy Robertson** (Western))

JULIE BERGNER, University of California, Riverside
Homotopy operads as diagrams

Many algebraic structures on spaces can be encoded via product-preserving functors from an algebraic theory to the category of spaces. For some structures, the algebraic theory can be replaced by a simpler diagram. For example, simplicial monoids are known to be equivalent to Segal monoids, given by certain Δ^{op}-diagrams. In joint work with Philip Hackney, we establish an equivalence of model categories between simplicial operads and certain Ω^{op}-diagrams, where Ω is the Moerdijk-Weiss dendroidal category. Furthermore, we extend this result to a diagrammatic description of simplicial operads with a group action.

MICHAEL CHING, Amherst College
A classification of Taylor towers

Goodwillie's homotopy calculus provides a systematic way to approximate a functor F between the categories of based spaces and/or spectra with a 'Taylor tower' of polynomial functors. The layers in this tower can be described by a sequence of spectra which play the role of the derivatives of F (at the one-point object). The goal of this talk is to describe additional structure on these derivatives that specifies the extensions in the Taylor tower. This allows us to describe the polynomial approximations as derived mapping objects for coalgebras over certain comonads. I'll connect the structure of these comonads to that of right modules over various operads. This is all joint work with Greg Arone.

MARTIN FRANKLAND, University of Illinois at Urbana-Champaign
Non-realizable 2-stage Π-algebras

It is a classic fact that Eilenberg-MacLane spaces exist and are unique up to weak equivalence. However, one cannot always find a space with two non-zero homotopy groups and prescribed primary homotopy operations. Using work of Baues and Goerss, we will present examples of non-realizable 2-stage Π-algebras, focusing on the stable range.

PHIL HACKNEY, University of California, Riverside
Homotopy theory of props

Props have the capability to control algebraic structures more general than those described by operads; for example, there is a prop governing Hopf algebras and a prop governing conformal field theories. We study the category consisting of all (colored, simplicial) props. We show that this category is a closed symmetric monoidal category with tensor product closely related to the Boardman-Vogt tensor product of operads. Furthermore, this category admits a Quillen model structure which restricts to the model structure for (colored) operads developed by Robertson and to the Bergner model structure for simplicial categories. (joint with Marcy Robertson)

RICK JARDINE, University of Western Ontario
Cosimplicial spaces and cocycles

Cosimplicial spaces were introduced by Bousfield and Kan in the early 1970s as a technical device in their theory of homology completions. These objects have since become fundamental tools in much of homotopy theory, but the original theory remains rather mysterious. The point of this talk is that cosimplicial spaces are quite amenable to study with modern methods of sheaf theoretic homotopy theory and cocycle categories. Non-abelian cohomology theory has a particularly interesting and useful interpretation in this context.

BRENDA JOHNSON, Union College

Models for Taylor Polynomials of Functors

Let $\mathcal{C}$ and $\mathcal{D}$ be simplicial model categories. Let $f : A \to B$ be a fixed morphism in $\mathcal{C}$ and $\mathcal{C}_f$ be the category whose objects are pairs of morphisms $A \to X \to B$ in $\mathcal{C}$ that factor f. Using a generalization of Eilenberg and Mac Lane's notion of cross effect functors (originally defined for functors of abelian categories) to functors from $\mathcal{C}_f$ to $\mathcal{D}$, we produce a tower of functors, $\cdots \to \Gamma_n^f F \to \Gamma_{n-1}^f F \to \cdots \to \Gamma_0^f F$, that acts like a Taylor series for the functor F. We compare this to the Taylor tower for F produced by Tom Goodwillie's calculus of homotopy functors, and use it to better understand the roles of the initial and final objects, A and B, in the calculus of homotopy functors. This is joint work with Kristine Bauer, Rosona Eldred, and Randy McCarthy.

KEITH JOHNSON, Dalhousie University

Homogeneous integer valued polynomials and the stable homotopy of BU

The use of homogeneous integer valued multivariable polynomials to detect elements in the stable homotopy groups of BU originated with work of Baker, Clarke, Ray and Schwartz (Trans. AMS 316(1989)). In this talk we will demonstrate some new constructions of rings of such polynomials and study their topological consequences.

DAN LIOR, University of Illinois, Urbana

The use of labelled trees in the Goodwillie-Taylor tower of discrete modules

A discrete module is a functor of finite pointed sets taking values in chain complexes of abelian groups. For an arbitrary discrete module F, McCarthy, Johnson and Intermont described the first homogeneous layer D_1F of the Goodwillie-Taylor tower of F in terms of the cross effects of F and the multilinear parts of finitely generated free Lie algebras. I will describe a category of trees which illustrates this connection and extends it to the rest of the layers D_nF of the Goodwillie-Taylor tower of F.

PARKER LOWREY, University of Western Ontario

The derived motivic Hall algebra associated to a projective variety.

We discuss how to associate a locally geometric derived moduli stack classifying objects in the bounded derived category associated to any projective variety. This is the main ingredient needed in defining a Hall algebra for this triangulated category. It extends the work of Toën, Kontsevich, and Soibelman to the singular case and is the first step in applying Donaldson-Thomas theory (and Joycey's extensions) to these homologically unwieldy categories. The talk will contain a good deal of algebro-geometric and homotopy theoretic material.

HUGO RODRIGUEZ ORDONEZ, Universidad Autónoma de Aguascalientes

Dimensional restrictions upon counterexamples to Ganea's conjecture

The long standing conjecture by Ganea on the Lusternik-Schnirelmann category was disproved in the late 1990s by means of a family of counterexamples whose least dimensional element has dimension 10. In a previous work, the authors proved that there is a 7-dimensional counterexample. In this work, we present a proof that there is no counterexample to this conjecture with dimension 6 or less. This is joint work with Don Stanley.

SIMONA PAOLI, University of Leicester

n-fold groupoids and n-types

Most homotopy invariants of topological spaces are filtered by dimension, so it is useful to have finite dimensional approximations to homotopy theories. We describe an algebraic model for the latter, which we call n-track categories. An appropriate algebraic model of n-types is developed for this purpose, with a class of n-fold groupoids which we call n-typical. This model leads to an explicit connection between homotopy types and iterated loop spaces and exhibit other useful properties. This is joint work with David Blanc.

DORETTE PRONK, Dalhousie University
Bredon Cohomology with Local Coefficients

Bredon [1] defined his version of equivariant cohomology with constant coefficients for spaces with an action of a discrete group G. This was generalized to arbitrary topological groups by Illman [2]. This definition was then extended to local coefficient systems independently by Moerdijk and Svensson [3] and by the Mukherjees [4]. Moerdijk and Svensson's approach was only applicable to discrete groups and used the cohomology of a category constructed to represent the G-space. The Mukherjees' approach was closer to the work by Illman. Mukherjee and Pandey [5] showed that the two definitions agree when the group G is discrete.

Laura Scull and I have generalized the construction of the category given by Moerdijk and Svensson to G-spaces for an arbitrary toplogical group G. We will show that the resulting definition of Bredon cohomology agrees with the one given by the Mukherjees. As an application we get the Serre spectral sequence in the more general setting of a topological group G.

[1] G.E. Bredon, *Introduction to Compact Transformation Groups*, Academic Press (1972).

[2] S. Illman, Equivariant Singular Homology and Cohomology, Bull. AMS 79 (1973) pp. 188–192.

[3] I. Moerdijk, J.-A. Svensson, The equivariant Serre spectral sequence, *Proceedings of the AMS* 118 (1993), pp. 263–278.

[4] A. Mukherjee, G. Mukherjee, Bredon-Illman cohomology with local coefficients, *Quart. J. Math. Oxford* 47 (1996), pp. 199-219.

[5] Goutam Mukherjee, Neeta Pandey, Equivariant cohomology with local coefficients, *Proceedings of the AMS* 130 (2002), pp. 227-232.

MARCY ROBERTSON, University of Western Ontario
On Topological Triangulated Orbit Categories

In 2005, Keller showed that the orbit category associated to the bounded derived category of a hereditary category under an auto equivalence is triangulated. As an application he proved that the cluster category is triangulated. We show that this theorem generalizes to triangulated categories with topological origin (i.e. the homotopy category of a stable model category). As an application we construct a topological triangulated category which models the cluster category. This is joint work with Andrew Salch.

DON STANLEY, University of Regina
Homotopy invariance of configuration spaces

Given a closed manifold M, the configuration space of n points in M, $F(M,k)$ is the set k distinct points in M. Levitt showed that if M is 2-connected then $F(M,2)$ only depends on the homotopy type of M. When M is a smooth projective variety, Kriz constructed a model for the rational homotopy type of $F(M,k)$. In this talk we show that a variant of the Kriz model works for any sufficiently connected closed manifold, and discuss the related problem of the homotopy invariance of $F(M,3)$.

SEAN TILSON, Wayne State University
Power Operations and the Kunneth Spectral Sequence

Power operations have been constructed and successfully utilized in the Adams and Homological Homotopy Fixed Point Spectral Sequences by Bruner and Bruner-Rognes. It was thought that such results were not specific to the spectral sequence, but rather that they arose because highly structured ring spectra are involved. In this talk, we show that while the Kunneth Spectral Sequence enjoys some nice multiplicative properties, there are no non-zero algebraic operations in E_2 (other than the square).

Despite the negative results we are able to use old computations of Steinberger's with our current work to compute operations in the homotopy of some relative smash products.