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Random matrices: invertibility, structure, and applications

At the heart of random matrix theory lies the realization that the spectrum of a random matrix H tends to stabilize as the dimensions of H grow to infinity. This phenomenon is captured by the limit laws of random matrix theory, in particular by Wigner's semicircle law, Girko's circular law, and Marchenko-Pastur law. These limit laws offer us a clear global and asymptotic picture of the spectrum of H.

In the last few years, a considerable progress was made on the more difficult local and non-asymptotic regimes. In the non-asymptotic regime, the dimensions of H are fixed rather than grow to infinity. In the local regime, one zooms in on a small part of the spectrum of H until one sees individual eigenvalues. The location of the eigenvalue nearest zero determines the invertibility properties of H. This essentially determines whether the matrix H is well conditioned, which is a matter of importance in numerical analysis.

Examples of recent developments include the proofs that a random matrix H with independent entries (whether symmetric or not) is singular with an exponentially small probability, that the condition number of H is linear in the dimension, and that the eigenstructure of H is delocalized and unstructured – the eigenvectors are spread out and their coefficients are highly incommensurate.

We will see some examples of heuristics, results, and problems of the non-asymptotic random matrix theory in the local regime. Applications and problems in related areas will be discussed, in particular for covariance estimation in statistics.