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Differential Categories to Tangential Structure

In this talk I describe work with Goeff Cruttwell and Johnathan Gallagher on differential categories and, in particular, on the passage from differential to tangential restriction categories.

The study of abstract differential structure for categories was initiated by Thomas Ehrhard who discovered a number of models of differential structures arising from his work in linear logic. These are interesting models which have computational content which appears to be related to the semantics of distributed systems.

This work exposed the purely algebraic structure underlying differentiation. In 2008, together with Robert Seely and Rick Blute, we introduced Cartesian differential categories. An important way in which these arise is as the coKleisli category of a models of differential linear logic (as mentioned above). However, significantly, they also arise much more directly from standard (algebraic and synthetic) models of the ordinary differential calculus (on the real line).

Given these of purely algebraic/categorical descriptions of differentiation, the possibility of capturing abstractly some of the basic ideas of differential geometry (e.g. smooth manifolds etc.) came within reach. To this end we introduced the notion of a differential restriction category: this adds partiality (and therefore topology) to the differential structure. From there one can use the manifold construction of Marco Grandis to obtain categories of "smooth manifolds". Furthermore, one can axiomatize the structure which arises using tangent spaces – here called tangential structure.

An important example of all this structure, which I will discuss, is central to algebraic geometry.