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A new construction of compact G_2 manifolds by glueing families of Eguchi-Hanson spaces

Manifolds with G_2 holonomy are examples of Ricci-flat Riemannian manifolds with parallel spinors, and play a role in M-theory analogous to the role of Calabi-Yau manifolds in string theory. They are 7-dimensional manifolds equipped with a "non-degenerate" 3-form φ which determines a metric g_φ and an orientation in a highly nonlinear way, and such that φ is parallel with respect to the Levi-Civita connection of g_φ. Until now, there have been only two known constructions of compact examples, due to Joyce (1994) and Kovalev (2000), both using glueing techniques and the Fredholm theory of elliptic nonlinear PDE's on manifolds.

In this talk, I will discuss a new glueing-type construction (joint work with Dominic Joyce) of compact G_2 manifolds. This construction involves glueing a family of Eguchi-Hanson spaces along a special Lagrangian submanifold L of a Calabi-Yau manifold X that admits an antiholomorphic isometric involution, in a particular way. We then show that one can obtain a G_2 holonomy metric on a particular compact smooth 7-manifold M obtained from X, L, and the Eguchi-Hanson spaces. The main difficulty is that there is no natural torsion-free G_2-structure on this varying family of Eguchi-Hanson spaces, and as a result in order to control the size of the error terms we need to solve an elliptic equation on the noncompact Eguchi-Hanson space to find a suitable correction term.